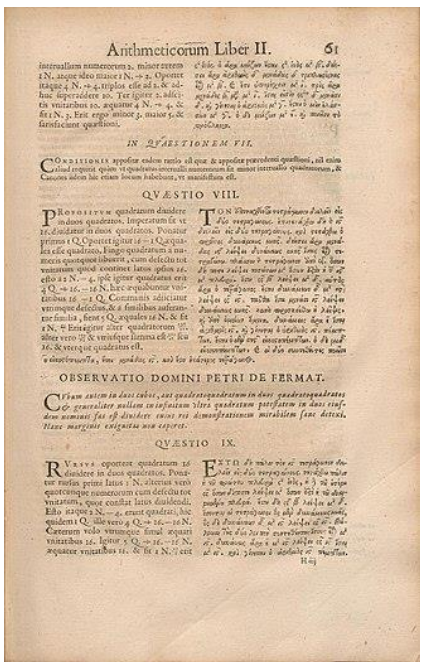
What was Fermat Thinking?

On FLT* and Its Inverse (FLT^{-1}), Algebraic Families, and Haeckel's Mantra ("*Ontogeny Recapitulates Phylogeny*")


Arithmeticorum Liber II. 61

IN QVÆSTIONEM VII.

QVÆSTIO VIII.

OBSERVATIO DOMINI PETRI DE FERMAT.

QVÆSTIO IX.

it is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain.

In other words

$$a^n + b^n = c^n$$

does not have rational, non-trivial solutions for n > 2.

*Fermat's Last Theorem (FLT)

Preface

In 1866 Ernst Haeckel hypothesized that the embryonal development of advanced species passes through stages represented by organisms of more primitive species. Otherwise put, each successive stage in the development of an individual represents one of the prior forms that appeared in its evolutionary history. Might the DNA of higher organisms then maintain that of prior ancestral prototypes?

Mathematical expressions known as equations which have identical *ranges* or values – but different equidistant *roots* - can be represented as *families*. These families have constant terms which specifically recapitulate prior expressions as well as values.

Human embryogenesis, as understood today, apparently involves the coordinated activity of many different genes located on various chromosomes. Much of the total genome contains introns which do not represent amino acids and whose function is yet unclear. Might it be conceivable that therein pointers are targeted in stem cells - as well as cells later in differentiation - which specify other specific genetic pathways on the same or different chromosomes?

If so, might there be a mathematical connection? What about the Fibonacci series and its prevalence in biologic structures? What might be the relationship?

Wo sind die fragen ungefracht und unbeantwortet? -DM Delinferni, Jr.

(*Vielleicht* – "**Why is it asking how n>2 *does* cause the problem - rather than questioning how n=2 uniquely and differently *does not*?**")

Step 1: Show that the set of of all rationals excluding {0} to any specific power forms a group (*n-powers*) under multiplication (*trivial*)

Step 2: Show that the set of every difference between the members of each *n-power* group forms another group (*n-diffs*) under both

A: Diagonal addition where $(\mathbf{R_1, C_1}) + (\mathbf{R_1 + C_1, C_2}) = (\mathbf{R_1, C_1 + C_2})$ When, for example, the *3-diffs* are represented by the following (**R**ows, **C**olumns):-

Row = v	COLUMN=>	1	2	3	4	5	6	7
0	0	1	8	27	64	125	216	343
1	1	7	26	63	124	215	342	511
2	8	19	56	117	208	335	504	721
3	27	37	98	189	316	485	702	973
4	64	61	152	279	448	665	936	1267
5	125	91	218	387	604	875	1206	1603
6	216	127	296	513	784	1115	1512	1981
7	343	169	386	657	988	1385	1854	2401
8	512	217	488	819	1216	1685	2232	2863
9	729	271	602	999	1468	2015	2646	3367
10	1000	331	728	1197	1744	2375	3096	3913
11	1331	397	866	1413	2044	2765	3582	4501
12	1728	469	1016	1647	2368	3185	4104	5131
13	2197	547	1178	1899	2716	3635	4662	5803
14	2744	631	1352	2169	3088	4115	5256	6517
15	3375	721	1538	2457	3484	4625	5886	7273
16	4096	817	1736	2763	3904	5165	6552	8071
17	4913	919	1946	3087	4348	5735	7254	8911
18	5832	1027	2168	3429	4816	6335	7992	9793
19	6859	1141	2402	3789	5308	6965	8766	10717
20	8000	8000	1261	2648	4167	5824	7625	9576

B: and Multiplication (excluding 0) by the respective *n-power* group (abelian).

Step 3: Show that the intersection of the above two groups for each power is {0,1,-1} excepting ***iff*** the following holds (for any base **x** where $\mathbf{X^n+Y^n=Z^n}$ and **a=Z-Y**):-

$$(2a \cdot a^2{}_{x0})^n + (2a \cdot 2a^2 \cdot (a^3 - a)/2_{x0})^n = (2a \cdot 2a^2 \cdot (a^3 + a)/2_{x0})^n$$

or, when a=1,

$$k((21_{x1})^n + (220_{x1})^n = (221_{x1})^n)$$

Step 4: Show that that the above only holds for n=2 (!) when $k=a^{-1}$ and $x_1=x_0+(a-1)/2$ (see Addendum 3)

It is universally true, i.e., including the rationals, for any base **x**, that

$$k((21_x)^2 + (220_x)^2 = (221_x)^2)$$

Therefore, it is also holds that in order for **$(221_x) - (220_x) = 1$** when **$X^n = Z^n - Y^n$**, that

$$k = ((Z^n)^{1/2}) - ((Y^n)^{1/2})$$

and

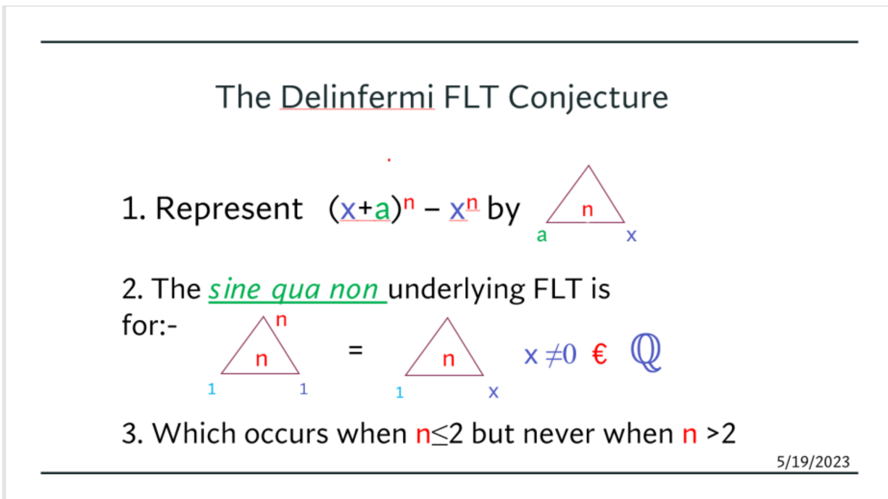
$$((X^n)^{1/2}) = 21_x/k,$$
$$x= k(((X^n)^{1/2})- 1)/2$$
$$k((Z^n)^{1/2}) = 221_x,$$
$$k((Y^n)^{1/2}) = 220_x$$

---

1. Step 5: Prove the Delinferni FLT Conjecture by showing that the only solutions to n>2 exclude the rationals completely. Since FLT has been proven for all even powers (2n | n integer >1) then:-

   $$((2a \cdot a^2{}_x)^{1/n})^{2n} + ((2a \cdot 2a^2 \cdot (a^3 - a)/2_x)^{1/n})^{2n} = ((2a \cdot 2a^2 \cdot (a^3 + a)/2_x)^{1/n})^{2n}$$

   Or $k(((21_x)^{1/n})^{2n} + ((220_x)^{1/n})^{2n} = ((221_x)^{1/n})^{2n})$ **proves FLT for all n=2n+1 (for all odd n integers odd) and therefore for all n-integers >2**



Step 6: Show that most integral squares do not fit into the Pythagorean squares but do so if and when and only if and when the restrictions in Step 4 apply

Step 7: Show that when $x^n + y^n = z^n$, there exists a series of logical tranformations of

$x \to x_n$, $y \to y_n$, and $z \to z_n$, and constant k such that

$$\mathbf{k((2x_n+1)^2 + (2x_n^2+2x_n)^2 = (2x_n^2+2x_n+1)^2}$$

and with k and x_n non-rational when n>2.

For example:

$$13^2 = 8^3 – 7^3 \text{ as } 512-343=169$$

$$13^2 =(\sqrt[2]{512})^2 - (\sqrt[2]{343})^2$$

$$k = (\sqrt[2]{512}) - (\sqrt[2]{343})$$

$$k=22.62741699797\ldots – 18.520259177452136\ldots = \mathbf{4.10757820517\ldots}$$

$$13^2/4.10757820517\ldots = (221_x)^2 - (220_x)^2 = 3.1648819208451244\ldots$$

$$221_x = \sqrt[2]{512})$$

$$220_x = \sqrt[2]{342})$$

$$3.1648819208451244\ldots^2 = 5.508702663162933\ldots^2 - 4.57405636\ldots^2$$

$2x+1 = 3.1648819208451244\ldots$ (=13/4.10757820517…)

x=**1.0824409604225622…**

$$\mathbf{4.10757820517\ldots*(21_{1.08244\ldots}{}^2 + 220_{1.08244\ldots}{}^2 = 221_{1.08244\ldots}{}^2)}$$

Step 8: Show that the solutions for n=2 are only enabled because

$$\mathbf{(a^{n1})^{2\backslash}+((a^{n2}-a^{n3})/2)^2 = ((a^{n2}+a^{n3})/2)^2}$$

applies as long as n1=(n2+n3)/2, but that the additional terms do not cancel out when n1>2

Step 9: Show since the universal expression for the sum of powers differing by one is:

$\mathbf{(x-1)^n + x^n = (x+1)^n}$, its universal solution is:-

$\mathbf{(2n+1+\ln(n/2))^n + (2n+\ln(n/2))^n = (2n+1+\ln(n/2))^n}$ and, by Step 7:

$$\left(\sqrt[2]{\left(\mathbf{2n-1+\ln\left(\tfrac{n}{2}\right)}\right)^n}\right)^2 + \left(\sqrt[2]{\left(\mathbf{2n-1+\ln\left(\tfrac{n}{2}\right)}\right)^n}\right)^2 = \left(\sqrt[2]{\left(\mathbf{2n+1+\ln\left(\tfrac{n}{2}\right)}\right)^n}\right)^2$$

And, therefore,if rational solutions exist for x=1, then

$$k(2x+1)= \left(\sqrt[2]{\left(\mathbf{2n-1+\ln\left(\tfrac{n}{2}\right)}\right)^n}\right),$$

$$x=\left(\left(\sqrt[2]{\left(\mathbf{2n-1+\ln\left(\tfrac{n}{2}\right)}\right)^n}\right) - k\right)/2k = 2^n-1, \text{ and}$$

$$k=\left(\sqrt[2]{\left(\mathbf{2n-1+\ln\left(\tfrac{n}{2}\right)}\right)^n}\right)/(2^{n+1}-1) \text{ which is only rational for } n<=2$$

[Q.E.D]

Step 10: Show that there exists an inverse representation (**FLT^{-1}**) which only works when n>2 for a similar rationale and that is:-

$$\mathbf{a^2 + b^n = (a+b)^2}$$

I. **Margins?**

If one makes tables of powers by increasing integers then a remarkable group symmetry seems to exist across the tables. This symmetry also appears to be preserved among the differences between these powers. That symmetry is obtained in one table simply by dividing x^n by $(x-y)^n$ - which maintains the same power structure - and in the other table by dividing $(x-y)^n$ by the same z^n-y^n or:

$$(z^n - y^n)/(z-y)^n = x^n /(z-y)^n = ((z-y)^n/ x^n)^{-1}$$

Initially these tables will be shown for n-3, but they are followed by tables for n=2 and 5. Perhaps it maight require some extremely large margins for these tables to be placed inside them?

Nevertheless, as usual a graphic representation leads to an interesting discovery.

I will leave it to the reader to see if he or she can arrive at the same conclusion after studying these tables

n=3	$(z^n - y^n)/(z-y)^n$			$((Row\# + Col\#)^3-Row\#^3)/Col\#^3$															
Row#=n	n^3	Col# -5	Col# -4	Col# -3	Col# -2	Col# -1	Col #0	Col #1	Col #2	Col #3	Col #4	Col #5	Col #6	Col #7	Col #8	Col #9	Col #10	#11	Col #12
-10	-1000						(Row#, Col#)	271	61	24.33333	12.25	7	4.333333	2.836735	1.9375	1.37037	1	0.752066	0.583333
-9	-729					271		217	48.25	19	9.4375	5.32	3.25	2.102041	1.421875	1	0.73	0.553719	0.4375
-8	-512					217		169	37	14.33333	7	3.88	2.333333	1.489796	1	0.703704	0.52	0.404959	0.333333
-7	-343				48.25	169		127	27.25	10.33333	4.9375	2.68	1.583333	1	0.671875	0.481481	0.37	0.305785	0.270833
-6	-216			19	37	127		91	19	7	3.25	1.72	1	0.632653	0.4375	0.333333	0.28	0.256198	0.25
-5	-125	7	9.4375	14.33333	27.25	91		61	12.25	4.333333	1.9375	1	0.583333	0.387755	0.296875	0.259259	0.25	0.256198	0.270833
-4	-64	5.32	7	10.33333	19	61		37	7	2.333333	1	0.52	0.333333	0.265306	0.25	0.259259	0.28	0.305785	0.333333
-3	-27	3.88	4.9375	7	12.25	37		19	3.25	1	0.4375	0.28	0.25	0.265306	0.296875	0.333333	0.37	0.404959	0.4375
-2	-8	2.68	3.25	4.333333	7	19	(-2,0)	7	1	0.333333	0.25	0.28	0.333333	0.387755	0.4375	0.481481	0.52	0.553719	0.583333
-1	-1	1.72	1.9375	2.333333	3.25	7	(-1,0)	1	0.25	0.333333	0.4375	0.52	0.583333	0.632653	0.671875	0.703704	0.73	0.752066	0.770833
0	0	1	1	1	1	1	(0,0)	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0.52	0.4375	0.333333	0.25	1	(1,0)	7	3.25	2.333333	1.9375	1.72	1.583333	1.489796	1.421875	1.37037	1.33	1.297521	1.270833
2	8	0.28	0.25	0.333333	1	7	(2,0)	19	7	4.333333	3.25	2.68	2.333333	2.102041	1.9375	1.814815	1.72	1.644628	1.583333
3	27	0.28	0.4375	1	3.25	19	(3,0)	37	12.25	7	4.9375	3.88	3.25	2.836735	2.546875	2.333333	2.17	2.041322	1.9375
4	64	0.52	1	2.333333	7	37		61	19	10.33333	7	5.32	4.333333	3.693878	3.25	2.925926	2.68	2.487603	2.333333
5	125	1	1.9375	4.333333	12.25	61		91	27.25	14.33333	9.4375	7	5.583333	4.673469	4.046875	3.592593	3.25	2.983471	2.770833
6	216	1.72	3.25	7	19	91		127	37	19	12.25	8.92	7	5.77551	4.9375	4.333333	3.88	3.528926	3.25
7	343	2.68	4.9375	10.33333	27.25	127		169	48.25	24.33333	15.4375	11.08	8.583333	7	5.921875	5.148148	4.57	4.123967	3.770833
8	512	3.88	7	14.33333	37	169		217	61	30.33333	19	13.48	10.33333	8.346939	7	6.037037	5.32	4.768595	4.333333
9	729	5.32	9.4375	19	48.25	217		271	75.25	37	22.9375	16.12	12.25	9.816327	8.171875	7	6.13	5.46281	4.9375
10	1000	7	12.25	24.33333	61	271		331	91	44.33333	27.25	19	14.33333	11.40816	9.4375	8.037037	7	6.206612	5.583333
11	1331	8.92	15.4375	30.33333	75.25	331		397	108.25	52.33333	31.9375	22.12	16.58333	13.12245	10.79688	9.148148	7.93	7	6.270833
12	1728	11.08	19	37	91	397		469	127	61	37	25.48	19	14.95918	12.25	10.33333	8.92	7.842975	7
13	2197	13.48	22.9375	44.33333	108.25	469		547	147.25	70.33333	42.4375	29.08	21.58333	16.91837	13.79688	11.59259	9.97	8.735537	7.770833
14	2744	16.12	27.25	52.33333	127	547		631	169	80.33333	48.25	32.92	24.33333	19	15.4375	12.92593	11.08	9.677686	8.583333
15	3375	19	31.9375	61	147.25	631		721	192.25	91	54.4375	37	27.25	21.20408	17.17188	14.33333	12.25	10.66942	9.4375
16	4096	22.12	37	70.33333	169	721		817	217	102.3333	61	41.32	30.33333	23.53061	19	15.81481	13.48	11.71074	10.33333
17	4913	25.48	42.4375	80.33333	192.25	817		919	243.25	114.3333	67.9375	45.88	33.58333	25.97959	20.92188	17.37037	14.77	12.80165	11.27083
18	5832	29.08	48.25	91	217	919		1027	271	127	75.25	50.68	37	28.55102	22.9375	19	16.12	13.94215	12.25
19	6859	32.92	54.4375	102.3333	243.25	1027		1141	300.25	140.3333	82.9375	55.72	40.58333	31.2449	25.04688	20.7037	17.53	15.13223	13.27083

n=3		$x^n/(z-y)^n$		$(Col\# + Row\#)^3/(Row\#)^3$														
Row#=n	n^3	Col# -4	Col# -3	Col# -2	Col# -1	Col# 0	Col #1	Col #2	Col #3	Col #4	Col #5	Col #6	Col #7	Col #8	Col #9	Col #10	Col #11	Col #12
-9	-729						0.702332	0.470508	0.296296	0.171468	0.087791	0.037037	0.010974	0.001372	0	-0.00137	-0.01097	-0.03704
-8	-512					1.423828125	0.669922	0.421875	0.244141	0.125	0.052734	0.015625	0.001953	0	-0.00195	-0.01563	-0.05273	-0.125
-7	-343				2.125364	1.49271137	0.629738	0.364431	0.186589	0.078717	0.023324	0.002915	0	-0.00292	-0.02332	-0.07872	-0.18659	-0.36443
-6	-216			3.375	2.37037	1.587962963	0.578704	0.296296	0.125	0.037037	0.00463	0	-0.00463	-0.03704	-0.125	-0.2963	-0.5787	-1
-5	-125		5.832	4.096	2.744	1.728	0.512	0.216	0.064	0.008	0	-0.008	-0.064	-0.216	-0.512	-1	-1.728	-2.744
-4	-64	11.39063	8	5.359375	3.375	1.953125	0.421875	0.125	0.015625	0	-0.01563	-0.125	-0.42188	-1	-1.95313	-3.375	-5.35938	-8
-3	-27	18.96296	12.7037	8	4.62963	2.37037037	0.296296	0.037037	0	-0.03704	-0.2963	-1	-2.37037	-4.62963	-8	-12.7037	-18.963	-27
-2	-8	42.875	27	15.625	8	3.375	0.125	0	-0.125	-1	-3.375	-8	-15.625	-27	-42.875	-64	-91.125	-125
-1	-1	216	125	64	27	8	0	-1	-8	-27	-64	-125	-216	-343	-512	-729	-1000	-1331
0	0																	
1	1	-64	-27	-8	-1	0	8	27	64	125	216	343	512	729	1000	1331	1728	2197
2	8	-3.375	-1	-0.125	0	0.125	3.375	8	15.625	27	42.875	64	91.125	125	166.375	216	274.625	343
3	27	-0.2963	-0.03704	0	0.037037	0.296296296	2.37037	4.62963	8	12.7037	18.96296	27	37.03704	49.2963	64	81.37037	101.6296	125
4	64	-0.01563	0	0.015625	0.125	0.421875	1.953125	3.375	5.359375	8	11.39063	15.625	20.79688	27	34.32813	42.875	52.73438	64
5	125	0	0.008	0.064	0.216	0.512	1.728	2.744	4.096	5.832	8	10.648	13.824	17.576	21.952	27	32.768	39.304
6	216	0.00463	0.037037	0.125	0.296296	0.578703704	1.587963	2.37037	3.375	4.62963	6.162037	8	10.1713	12.7037	15.625	18.96296	22.74537	27
7	343	0.023324	0.078717	0.186589	0.364431	0.629737609	1.492711	2.125364	2.915452	3.880466	5.037901	6.405248	8	9.83965	11.94169	14.32362	17.00292	19.99708
8	512	0.052734	0.125	0.244141	0.421875	0.669921875	1.423828	1.953125	2.599609	3.375	4.291016	5.359375	6.591797	8	9.595703	11.39063	13.39648	15.625
9	729	0.087791	0.171468	0.296296	0.470508	0.702331962	1.371742	1.825789	2.37037	3.013717	3.76406	4.62963	5.618656	6.739369	8	9.408779	10.97394	12.7037
10	1000	0.125	0.216	0.343	0.512	0.729	1.331	1.728	2.197	2.744	3.375	4.096	4.913	5.832	6.859	8	9.261	10.648
11	1331	0.162284	0.257701	0.384673	0.547708	0.751314801	1.298272	1.650639	2.061608	2.535687	3.077385	3.69121	4.381668	5.153268	6.010518	6.957926	8	9.141247
12	1728	0.198495	0.296296	0.421875	0.578704	0.77025463	1.271412	1.587963	1.953125	2.37037	2.843171	3.375	3.969329	4.62963	5.359375	6.162037	7.041088	8
13	2197	0.233045	0.331816	0.455166	0.605826	0.786527082	1.248976	1.536186	1.86436	2.236231	2.654529	3.121985	3.641329	4.215294	4.846609	5.538006	6.292217	7.111971
14	2744	0.265671	0.364431	0.485058	0.629738	0.800655977	1.229956	1.492711	1.790452	2.125364	2.499636	2.915452	3.375	3.880466	4.434038	5.037901	5.694242	6.405248
15	3375	0.296296	0.39437	0.512	0.650963	0.813037037	1.21363	1.455704	1.728	2.032296	2.37037	2.744	3.154963	3.605037	4.096	4.62963	5.207704	5.832
16	4096	0.324951	0.421875	0.536377	0.669922	0.823974609	1.199463	1.423828	1.674561	1.953125	2.260986	2.599609	2.970459	3.375	3.814697	4.291016	4.80542	5.359375
17	4913	0.35172	0.447181	0.558518	0.686953	0.833706493	1.187055	1.396092	1.628333	1.884999	2.167311	2.476491	2.813759	3.180338	3.577448	4.00631	4.468146	4.964177
18	5832	0.376715	0.470508	0.578704	0.702332	0.842421125	1.176097	1.371742	1.587963	1.825789	2.086248	2.37037	2.679184	3.013717	3.375	3.76406	4.181927	4.62963
19	6859	0.400058	0.492054	0.597172	0.716285	0.850269719	1.166351	1.350197	1.552413	1.773874	2.015454	2.278029	2.562473	2.86966	3.200467	3.555766	3.936434	4.343345
20	8000	0.421875	0.512	0.614125	0.729	0.857375	1.157625	1.331	1.520875	1.728	1.953125	2.197	2.460375	2.744	3.048625	3.375	3.723875	4.096

For n=2 square ratios:

1	1	4	9	16	25	36	49	64	81	100	121	144	169
2	4	2.25	4	6.25	9	12.25	16	20.25	25	30.25	36	42.25	49
3	9	1.777778	2.777778	4	5.444444	7.111111	9	11.11111	13.44444	16	18.77778	21.77778	25
4	16	1.5625	2.25	3.0625	4	5.0625	6.25	7.5625	9	10.5625	12.25	14.0625	16
5	25	1.44	1.96	2.56	3.24	4	4.84	5.76	6.76	7.84	9	10.24	11.56
6	36	1.361111	1.777778	2.25	2.777778	3.361111	4	4.694444	5.444444	6.25	7.111111	8.027778	9
7	49	1.306122	1.653061	2.040816	2.469388	2.938776	3.44898	4	4.591837	5.22449	5.897959	6.612245	7.367347
8	64	1.265625	1.5625	1.890625	2.25	2.640625	3.0625	3.515625	4	4.515625	5.0625	5.640625	6.25
9	81	1.234568	1.493827	1.777778	2.08642	2.419753	2.777778	3.160494	3.567901	4	4.45679	4.938272	5.444444
10	100	1.21	1.44	1.69	1.96	2.25	2.56	2.89	3.24	3.61	4	4.41	4.84
11	121	1.190083	1.396694	1.619835	1.859504	2.115702	2.38843	2.677686	2.983471	3.305785	3.644628	4	4.371901
12	144	1.173611	1.361111	1.5625	1.777778	2.006944	2.25	2.506944	2.777778	3.0625	3.361111	3.673611	4
13	169	1.159763	1.331361	1.514793	1.710059	1.91716	2.136095	2.366864	2.609467	2.863905	3.130178	3.408284	3.698225
14	196	1.147959	1.306122	1.47449	1.653061	1.841837	2.040816	2.25	2.469388	2.69898	2.938776	3.188776	3.44898
15	225	1.137778	1.284444	1.44	1.604444	1.777778	1.96	2.151111	2.351111	2.56	2.777778	3.004444	3.24
16	256	1.128906	1.265625	1.410156	1.5625	1.722656	1.890625	2.066406	2.25	2.441406	2.640625	2.847656	3.0625
17	289	1.121107	1.249135	1.384083	1.525952	1.67474	1.83045	1.99308	2.16263	2.3391	2.522491	2.712803	2.910035
18	324	1.114198	1.234568	1.361111	1.493827	1.632716	1.777778	1.929012	2.08642	2.25	2.419753	2.595679	2.777778
19	361	1.108033	1.221607	1.34072	1.465374	1.595568	1.731302	1.872576	2.019391	2.171745	2.32964	2.493075	2.66205
20	400	1.1025	1.21	1.3225	1.44	1.5625	1.69	1.8225	1.96	2.1025	2.25	2.4025	2.56
21	441	1.097506	1.199546	1.306122	1.417234	1.53288	1.653061	1.777778	1.907029	2.040816	2.179138	2.321995	2.469388
22	484	1.092975	1.190083	1.291322	1.396694	1.506198	1.619835	1.737603	1.859504	1.985537	2.115702	2.25	2.38843
23	529	1.088847	1.181474	1.277883	1.378072	1.482042	1.589792	1.701323	1.816635	1.935728	2.058601	2.185255	2.31569
24	576	1.085069	1.173611	1.265625	1.361111	1.460069	1.5625	1.668403	1.777778	1.890625	2.006944	2.126736	2.25
25	625	1.0816	1.1664	1.2544	1.3456	1.44	1.5376	1.6384	1.7424	1.8496	1.96	2.0736	2.1904
26	676	1.078402	1.159763	1.244083	1.331361	1.421598	1.514793	1.610947	1.710059	1.81213	1.91716	2.025148	2.136095
27	729	1.075446	1.153635	1.234568	1.318244	1.404664	1.493827	1.585734	1.680384	1.777778	1.877915	1.980796	2.08642
28	784	1.072704	1.147959	1.225765	1.306122	1.389031	1.47449	1.5625	1.653061	1.746173	1.841837	1.940051	2.040816
29	841	1.070155	1.142687	1.217598	1.294887	1.374554	1.456599	1.541023	1.627824	1.717004	1.808561	1.902497	1.998811
30	900	1.067778	1.137778	1.21	1.284444	1.361111	1.44	1.521111	1.604444	1.69	1.777778	1.867778	1.96
31	961	1.065557	1.133195	1.202914	1.274714	1.348595	1.424558	1.502601	1.582726	1.664932	1.74922	1.835588	1.924037
32	1024	1.063477	1.128906	1.196289	1.265625	1.336914	1.410156	1.485352	1.5625	1.641602	1.722656	1.805664	1.890625

and for n=2 square difference ratios (*note the multi-presence* ***<u>as well as the</u>*** ***<u>positions</u>*** *of actual squares*)

0	1	1	1	1	1	1	1	1	1	1	1.09009	1
1	3	2	1.666667	1.5	1.4	1.333333	1.285714	1.25	1.222222	1.2	1.288288	1.166667
4	5	3	2.333333	2	1.8	1.666667	1.571429	1.5	1.444444	1.4	1.486486	1.333333
9	7	4	3	2.5	2.2	2	1.857143	1.75	1.666667	1.6	1.684685	1.5
16	9	5	3.666667	3	2.6	2.333333	2.142857	2	1.888889	1.8	1.882883	1.666667
25	11	6	4.333333	3.5	3	2.666667	2.428571	2.25	2.111111	2	2.081081	1.833333
36	13	7	5	4	3.4	3	2.714286	2.5	2.333333	2.2	2.279279	2
49	15	8	5.666667	4.5	3.8	3.333333	3	2.75	2.555556	2.4	2.477477	2.166667
64	17	9	6.333333	5	4.2	3.666667	3.285714	3	2.777778	2.6	2.675676	2.333333
81	19	10	7	5.5	4.6	4	3.571429	3.25	3	2.8	2.873874	2.5
100	21	11	7.666667	6	5	4.333333	3.857143	3.5	3.222222	3	3.072072	2.666667
121	23	12	8.333333	6.5	5.4	4.666667	4.142857	3.75	3.444444	3.2	3.27027	2.833333
144	25	13	9	7	5.8	5	4.428571	4	3.666667	3.4	3.468468	3
169	27	14	9.666667	7.5	6.2	5.333333	4.714286	4.25	3.888889	3.6	3.666667	3.166667
196	29	15	10.33333	8	6.6	5.666667	5	4.5	4.111111	3.8	3.864865	3.333333
225	31	16	11	8.5	7	6	5.285714	4.75	4.333333	4	4.063063	3.5
256	33	17	11.66667	9	7.4	6.333333	5.571429	5	4.555556	4.2	4.261261	3.666667
289	35	18	12.33333	9.5	7.8	6.666667	5.857143	5.25	4.777778	4.4	4.459459	3.833333
324	37	19	13	10	8.2	7	6.142857	5.5	5	4.6	4.657658	4
361	39	20	13.66667	10.5	8.6	7.333333	6.428571	5.75	5.222222	4.8	4.855856	4.166667
400	41	21	14.33333	11	9	7.666667	6.714286	6	5.444444	5	5.054054	4.333333
441	0.5	22	15	11.5	9.4	8	7	6.25	5.666667	5.2	5.252252	4.5
484	0.5	23	15.66667	12	9.8	8.333333	7.285714	6.5	5.888889	5.4	5.45045	4.666667
529	0.5	24	16.33333	12.5	10.2	8.666667	7.571429	6.75	6.111111	5.6	5.648649	4.833333
576	0.5	25	17	13	10.6	9	7.857143	7	6.333333	5.8	5.846847	5
625	51	26	17.66667	13.5	11	9.333333	8.142857	7.25	6.555556	6	6.045045	5.166667
676	53	27	18.33333	14	11.4	9.666667	8.428571	7.5	6.777778	6.2	6.243243	5.333333
729	55	28	19	14.5	11.8	10	8.714286	7.75	7	6.4	6.441441	5.5
784	57	29	19.66667	15	12.2	10.33333	9	8	7.222222	6.6	6.63964	5.666667
841	59	30	20.33333	15.5	12.6	10.66667	9.285714	8.25	7.444444	6.8	6.837838	5.833333
900	61	31	21	16	13	11	9.571429	8.5	7.666667	7	7.036036	6
961	63	32	21.66667	16.5	13.4	11.33333	9.857143	8.75	7.888889	7.2	7.234234	6.166667

For n=5 ratios:

1	1	32	243	1024	3125	7776	16807	32768	59049	100000	161051	248832	371293
2	32	7.59375	32	97.65625	243	525.2188	1024	1845.281	3125	5032.844	7776	11602.91	16807
3	243	4.213992	12.86008	32	69.16461	134.8477	243	411.5226	662.7613	1024	1527.955	2213.267	3125
4	1024	3.051758	7.59375	16.41309	32	57.66504	97.65625	157.2764	243	362.5908	525.2188	741.5771	1024
5	3125	2.48832	5.37824	10.48576	18.89568	32	51.53632	79.62624	118.8138	172.1037	243	335.5443	454.3542
6	7776	2.161394	4.213992	7.59375	12.86008	20.71129	32	47.74859	69.16461	97.65625	134.8477	182.5948	243
7	16807	1.949664	3.513358	5.949902	9.582376	14.80526	22.09157	32	45.18207	62.38924	84.4801	112.4274	147.3255
8	32768	1.802032	3.051758	4.914886	7.59375	11.33096	16.41309	23.17429	32	43.3306	57.66504	75.56454	97.65625
9	59049	1.693509	2.727413	4.213992	6.28788	9.108097	12.86008	17.75773	24.0454	32	41.93295	54.19228	69.16461
10	100000	1.61051	2.48832	3.71293	5.37824	7.59375	10.48576	14.19857	18.89568	24.76099	32	40.84101	51.53632
11	161051	1.545051	2.305437	3.339464	4.715121	6.510832	8.816195	11.73273	15.37463	19.86948	25.35905	32	39.96463
12	248832	1.492143	2.161394	3.051758	4.213992	5.706087	7.59375	9.950887	12.86008	16.41309	20.71129	25.86622	32
13	371293	1.448516	2.045218	2.82412	3.824088	5.089156	6.668855	8.61853	10.99967	13.88023	17.33494	21.44566	26.30167
14	537824	1.41194	1.949664	2.640003	3.513358	4.603921	5.949902	7.59375	9.582376	11.96738	14.80526	18.15766	22.09157
15	759375	1.380841	1.869771	2.48832	3.260707	4.213992	5.37824	6.786676	8.475843	10.48576	12.86008	15.64626	18.89568
16	1048576	1.354081	1.802032	2.361392	3.051758	3.894902	4.914886	6.138175	7.59375	9.313226	11.33096	13.68418	16.41309
17	1419857	1.330816	1.743907	2.253748	2.876417	3.629684	4.533092	5.608046	6.877893	8.368009	10.10588	12.1212	14.44593
18	1889568	1.310405	1.693509	2.161394	2.727413	3.406251	4.213992	5.168179	6.28788	7.59375	9.108097	10.85494	12.86008
19	2476099	1.292355	1.649409	2.081351	2.599388	3.215794	3.943956	4.798425	5.794965	6.950598	8.283655	9.813824	11.5622
20	3200000	1.276282	1.61051	2.011357	2.48832	3.051758	3.71293	4.484033	5.37824	6.409734	7.59375	8.94661	10.48576
21	4084101	1.261877	1.575951	1.949664	2.391132	2.909178	3.513358	4.213992	5.022194	5.949902	7.009903	8.215867	9.582376
22	5153632	1.248895	1.545051	1.894901	2.305437	2.784232	3.339464	3.979941	4.715121	5.555141	6.510832	7.59375	8.816195
23	6436343	1.237135	1.517263	1.845982	2.229357	2.673936	3.186771	3.775436	4.448046	5.213276	6.080377	7.059199	8.160204
24	7962624	1.226433	1.492143	1.802032	2.161394	2.575928	3.051758	3.595442	4.213992	4.914886	5.706087	6.596051	7.59375
25	9765625	1.216653	1.469328	1.762342	2.100342	2.48832	2.931625	3.435974	4.007464	4.652587	5.37824	6.191736	7.100821
26	11881376	1.207681	1.448516	1.726328	2.045218	2.409582	2.82412	3.293843	3.824088	4.420521	5.089156	5.836357	6.668855
27	14348907	1.19942	1.429457	1.693509	1.995215	2.338466	2.727413	3.166473	3.66034	4.213992	4.8327	5.522035	6.28788
28	17210368	1.19179	1.41194	1.663483	1.949664	2.273943	2.640003	3.051758	3.513358	4.029197	4.603921	5.242433	5.949902
29	20511149	1.184722	1.395785	1.635912	1.908006	2.215157	2.56065	2.947966	3.380793	3.863029	4.398788	4.992407	5.64845
30	24300000	1.178154	1.380841	1.61051	1.869771	2.161394	2.48832	2.853661	3.260707	3.71293	4.213992	4.767745	5.37824
31	28629151	1.172037	1.366977	1.587034	1.834559	2.112049	2.422145	2.767639	3.15148	3.576774	4.046791	4.564971	5.134922
32	33554432	1.166326	1.354081	1.565274	1.802032	2.066611	2.361392	2.688891	3.051758	3.452784	3.894902	4.381193	4.914886

and for n=5 difference ratios:

1	1	31	7.5625	4.209877	3.050781	2.488	2.161265	1.949604	1.802002	1.693492	1.6105	1.545045	1.492139
2	32	211	31	12.7284	7.5625	5.368	4.209877	3.511454	3.050781	2.726871	2.488	2.305239	2.161265
3	243	781	90.0625	31	16.17578	10.408	7.5625	5.935444	4.907471	4.209877	3.7105	3.337955	3.050781
4	1024	2101	211	64.95062	31	18.568	12.7284	9.521449	7.5625	6.270538	5.368	4.708763	4.209877
5	3125	4651	427.5625	121.9877	54.61328	31	20.30941	14.61933	11.2356	9.055175	7.5625	6.491428	5.693528
6	7776	9031	781	211	90.0625	49.048	31	21.6289	16.17578	12.7284	10.408	8.767912	7.5625
7	16807	15961	1320.063	342.358	140.8633	74.248	45.58719	31	22.66138	17.4731	14.0305	11.62837	9.883343
8	32768	26281	2101	527.9136	211	108.328	64.95062	43.2324	31	23.49047	18.568	15.17116	12.7284
9	59049	40951	3187.563	781	304.9258	153.208	90.0625	58.87589	41.52856	31	24.1705	19.50283	16.17578
10	100000	61051	4651	1116.432	427.5625	211	121.9877	78.5302	54.61328	40.23945	31	24.73813	20.30941
11	161051	87781	6570.063	1550.506	584.3008	284.008	161.8835	102.8451	70.64966	51.46487	39.2305	31	25.21899
12	248832	122461	9031	2101	781	374.728	211	132.5202	90.0625	64.95062	49.048	38.41958	31
13	371293	166531	12127.56	2787.173	1023.988	485.848	270.6798	168.3053	113.3059	80.98933	60.6505	47.13619	37.75371
14	537824	221551	15961	3629.765	1320.063	620.248	342.358	211	140.8633	99.89194	74.248	57.29738	45.58719
15	759375	289201	20640.06	4651	1676.488	781	427.5625	261.454	173.2473	121.9877	90.0625	69.05888	54.61328
16	1048576	371281	26281	5874.58	2101	971.368	527.9136	320.5668	211	147.624	108.328	82.58459	64.95062
17	1419857	469711	33007.56	7325.691	2601.801	1194.808	645.1242	389.2882	254.6926	177.1667	129.2905	98.04665	76.72362
18	1889568	586531	40951	9031	3187.563	1454.968	781	468.6177	304.9258	211	153.208	115.6254	90.0625
19	2476099	723901	50250.06	11018.65	3867.426	1755.688	937.439	559.6047	362.3293	249.5261	180.3505	135.5093	105.1033
20	3200000	884101	61051	13318.28	4651	2101	1116.432	663.349	427.5625	293.1658	211	157.895	121.9877
21	4084101	1069531	73507.56	15961	5548.363	2495.128	1320.063	781	501.3137	342.358	245.4505	182.9876	140.8633
22	5153632	1282711	87781	18979.4	6570.063	2942.488	1550.506	913.7572	584.3008	397.56	284.008	211	161.8835
23	6436343	1526281	104040.1	22407.54	7727.113	3447.688	1810.032	1062.87	677.2708	459.2472	326.9905	242.1536	185.2074
24	7962624	1803001	122461	26281	9031	4015.528	2101	1229.638	781	527.9136	374.728	276.6779	211
25	9765625	2115751	143227.6	30636.8	10493.68	4651	2425.865	1415.411	896.2942	604.0712	427.5625	314.8105	239.432
26	11881376	2467531	166531	35513.47	12127.56	5359.288	2787.173	1621.587	1023.988	688.2504	485.848	356.7974	270.6798
27	14348907	2861461	192570.1	40951	13945.55	6145.768	3187.563	1849.617	1164.947	781	549.9505	402.8926	304.9258
28	17210368	3300781	221551	46990.88	15961	7016.008	3629.765	2101	1320.063	882.8869	620.248	453.3584	342.358
29	20511149	3788851	253687.6	53676.06	18187.74	7975.768	4116.606	2377.285	1490.259	994.4964	697.1305	508.4653	383.1704
30	24300000	4329151	289201	61051	20640.06	9031	4651	2680.071	1676.488	1116.432	781	568.492	427.5625
31	28629151	4925281	328320.1	69161.62	23332.74	10187.85	5235.958	3011.008	1879.732	1249.316	872.2705	633.7252	475.7398
32	33554432	5580961	371281	78055.32	26281	11452.65	5874.58	3371.796	2101	1393.788	971.368	704.4601	527.9136

and the same structure appears across all powers and their differences. For example, all red cells (ratio=2) for *n-powers* are 2^n but 2^n-1 for *n-diffs*.

And so, what is the amazing discovery that might, perhaps, have suggested itself to Pierre de Fermat? Well, it turns out that $(x^n + y^n)/(x-y)$ is completely divisible by x-y resulting in:

$$(z^n - y^n)/(z-y) = z^{n-1}+z^{n-2}y+\ldots+zy^{n-2}+y^{n-1}$$

and, therefore:

$$(z^n - y^n)=(z-y)(z^{n-1}+z^{n-2}y+\ldots+zy^{n-2}+y^{n-1})$$

and:

$$(z^n - y^n)/(z-y)^n = (z^{n-1}+z^{n-2}y+\ldots+zy^{n-2}+z^{n-1})/(z-y)^{n-1}$$

(See Addendum for examples of n=3.) Nevertheless, every $(z^{n-1}+z^{n-2}y+\ldots+zy^{n-2}+y^{n-1})$ for n>2 ***<u>has a non-zero discriminant</u>*** for z and or y, (and *its roots represent the non-trivial roots of unity when x or y =1*), and, it, therefore,

(1) cannot have any two roots the same
(2) cannot be evenly divisible by $(x-y)^{n-1}$, and
(3) $(z^{n-1}+z^{n-2}y+\ldots+zy^{n-2}+y^{n-1})/ (x-y)^{n-1}$ cannot be an n^{th} power of any rational number when n>2. Why not? It can be shown for ***all*** of the non-trivial roots of unity that:-
   a. $(nx+1)^{n-1}+ (nx+1)^{n-2}+\ldots+(nx+1)+1 = (n^{-1})*((nx+1)^n – (nx)^n-1)$ or
   b. $(1\cdot1\cdot1\cdot\ldots\cdot1\cdot1\ _{nx\cdot1}) = (n^{-1})*(n\cdot1_x)^n – (n\cdot0)_x{}^n – 1)$
(4) Unless z=y which is clearly unallowed as it represents division by zero and results in a trivial solution or z=-y which also results in a trivial solution
(5) (*Vielleicht* – "**Why is it asking how n>2 does cause the problem rather than discovering how n=2 uniquely differently does not?**")

In the case where n=2, then the equivalence reduces to:-

$$x^2 = (z+y)(z-y) = z^2-y^2$$

and $(z^2-y^2/(z-y) = (z+y)$

and $x^2/(z-y)^2 = (z+y)/(z-y)$

which clearly has infinitely many integral solutions.

Again, the differences among squares are pictured:

	0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	3	2	1.666667	1.5	1.4	1.333333	1.285714	1.25	1.222222	1.2	1.181818	1.166667
2	4	5	3	2.333333	2	1.8	1.666667	1.571429	1.5	1.444444	1.4	1.363636	1.333333
3	9	7	4	3	2.5	2.2	2	1.857143	1.75	1.666667	1.6	1.545455	1.5
4	16	9	5	3.666667	3	2.6	2.333333	2.142857	2	1.888889	1.8	1.727273	1.666667
5	25	11	6	4.333333	3.5	3	2.666667	2.428571	2.25	2.111111	2	1.909091	1.833333
6	36	13	7	5	4	3.4	3	2.714286	2.5	2.333333	2.2	2.090909	2
7	49	15	8	5.666667	4.5	3.8	3.333333	3	2.75	2.555556	2.4	2.272727	2.166667
8	64	17	9	6.333333	5	4.2	3.666667	3.285714	3	2.777778	2.6	2.454545	2.333333
9	81	19	10	7	5.5	4.6	4	3.571429	3.25	3	2.8	2.636364	2.5
10	100	21	11	7.666667	6	5	4.333333	3.857143	3.5	3.222222	3	2.818182	2.666667
11	121	23	12	8.333333	6.5	5.4	4.666667	4.142857	3.75	3.444444	3.2	3	2.833333
12	144	25	13	9	7	5.8	5	4.428571	4	3.666667	3.4	3.181818	3
13	169	27	14	9.666667	7.5	6.2	5.333333	4.714286	4.25	3.888889	3.6	3.363636	3.166667
14	196	29	15	10.33333	8	6.6	5.666667	5	4.5	4.111111	3.8	3.545455	3.333333
15	225	31	16	11	8.5	7	6	5.285714	4.75	4.333333	4	3.727273	3.5
16	256	33	17	11.66667	9	7.4	6.333333	5.571429	5	4.555556	4.2	3.909091	3.666667
17	289	35	18	12.33333	9.5	7.8	6.666667	5.857143	5.25	4.777778	4.4	4.090909	3.833333
18	324	37	19	13	10	8.2	7	6.142857	5.5	5	4.6	4.272727	4
19	361	39	20	13.66667	10.5	8.6	7.333333	6.428571	5.75	5.222222	4.8	4.454545	4.166667
20	400	41	21	14.33333	11	9	7.666667	6.714286	6	5.444444	5	4.636364	4.333333
21	441	43	22	15	11.5	9.4	8	7	6.25	5.666667	5.2	4.818182	4.5
22	484	45	23	15.66667	12	9.8	8.333333	7.285714	6.5	5.888889	5.4	5	4.666667
23	529	47	24	16.33333	12.5	10.2	8.666667	7.571429	6.75	6.111111	5.6	5.181818	4.833333
24	576	49	25	17	13	10.6	9	7.857143	7	6.333333	5.8	5.363636	5
25	625	51	26	17.66667	13.5	11	9.333333	8.142857	7.25	6.555556	6	5.545455	5.166667
26	676	53	27	18.33333	14	11.4	9.666667	8.428571	7.5	6.777778	6.2	5.727273	5.333333
27	729	55	28	19	14.5	11.8	10	8.714286	7.75	7	6.4	5.909091	5.5
28	784	57	29	19.66667	15	12.2	10.33333	9	8	7.222222	6.6	6.090909	5.666667
29	841	59	30	20.33333	15.5	12.6	10.66667	9.285714	8.25	7.444444	6.8	6.272727	5.833333
30	900	61	31	21	16	13	11	9.571429	8.5	7.666667	7	6.454545	6
31	961	63	32	21.66667	16.5	13.4	11.33333	9.857143	8.75	7.888889	7.2	6.636364	6.166667
32	1024	65	33	22.33333	17	13.8	11.66667	10.14286	9	8.111111	7.4	6.818182	6.333333
33	1089	67	34	23	17.5	14.2	12	10.42857	9.25	8.333333	7.6	7	6.5
34	1156	69	35	23.66667	18	14.6	12.33333	10.71429	9.5	8.555556	7.8	7.181818	6.666667
35	1225	71	36	24.33333	18.5	15	12.66667	11	9.75	8.777778	8	7.363636	6.833333
36	1296	73	37	25	19	15.4	13	11.28571	10	9	8.2	7.545455	7
37	1369	75	38	25.66667	19.5	15.8	13.33333	11.57143	10.25	9.222222	8.4	7.727273	7.166667
38	1444	77	39	26.33333	20	16.2	13.66667	11.85714	10.5	9.444444	8.6	7.909091	7.333333
39	1521	79	40	27	20.5	16.6	14	12.14286	10.75	9.666667	8.8	8.090909	7.5
40	1600	81	41	27.66667	21	17	14.33333	12.42857	11	9.888889	9	8.272727	
41	1681	83	42	28.33333	21.5	17.4	14.66667	12.71429	11.25	10.11111	9.2		
42	1764	85	43	29	22	17.8	15	13	11.5	10.33333			
43	1849	87	44	29.66667	22.5	18.2	15.33333	13.28571	11.75				
44	1936	89	45	30.33333	23	18.6	15.66667	13.57143					
45	2025	91	46	31	23.5	19	16						
46	2116	93	47	31.66667	24								
47	2209	95	48	32.33333	24.5								
48	2304	97	49	33	25								

Note that each cell represents a given numerator ratio which is repeated among the similarly-colored codes, i.e., **5:3 representing *4* in red**, **4:3 representing *9* in green**, **13:12 representing *25* in yellow-brown**, **37:35 representing 36 in pink**, **etc**. Since this structure appears to be universally-symmetric across all powers, then one should expect to find any higher integral powers somewhere among similarly placed cells.

In the next figure the squares are highlighted in yellow and the 2^n symmetry row in red:-

	0	1	1	1	1	1	1	1	1	1	1	1	1	1			
1	1	3	2	1.666667	1.5	1.4	1.333333	1.285714	1.25	1.222222	1.2	1.181818	1.166667	1.153846	1.142857	1.133333	1.125
2	4	5	3	2.333333	2	1.8	1.666667	1.571429	1.5	1.444444	1.4	1.363636	1.333333	1.307692	1.285714	1.266667	1.25
3	9	7	4	3	2.5	2.2	2	1.857143	1.75	1.666667	1.6	1.545455	1.5	1.461538	1.428571	1.4	1.375
4	16	9	5	3.666667	3	2.6	2.333333	2.142857	2	1.888889	1.8	1.727273	1.666667	1.615385	1.571429	1.533333	1.5
5	25	11	6	4.333333	3.5	3	2.666667	2.428571	2.25	2.111111	2	1.909091	1.833333	1.769231	1.714286	1.666667	1.625
6	36	13	7	5	4	3.4	3	2.714286	2.5	2.333333	2.2	2.090909	2	1.923077	1.857143	1.8	1.75
7	49	15	8	5.666667	4.5	3.8	3.333333	3	2.75	2.555556	2.4	2.272727	2.166667	2.076923	2	1.933333	1.875
8	64	17	9	6.333333	5	4.2	3.666667	3.285714	3	2.777778	2.6	2.454545	2.333333	2.230769	2.142857	2.066667	2
9	81	19	10	7	5.5	4.6	4	3.571429	3.25	3	2.8	2.636364	2.5	2.384615	2.285714	2.2	2.125
10	100	21	11	7.666667	6	5	4.333333	3.857143	3.5	3.222222	3	2.818182	2.666667	2.538462	2.428571	2.333333	2.25
11	121	23	12	8.333333	6.5	5.4	4.666667	4.142857	3.75	3.444444	3.2	3	2.833333	2.692308	2.571429	2.466667	2.375
12	144	25	13	9	7	5.8	5	4.428571	4	3.666667	3.4	3.181818	3	2.846154	2.714286	2.6	2.5
13	169	27	14	9.666667	7.5	6.2	5.333333	4.714286	4.25	3.888889	3.6	3.363636	3.166667	3	2.857143	2.733333	2.625
14	196	29	15	10.33333	8	6.6	5.666667	5	4.5	4.111111	3.8	3.545455	3.333333	3.153846	3	2.866667	2.75
15	225	31	16	11	8.5	7	6	5.285714	4.75	4.333333	4	3.727273	3.5	3.307692	3.142857	3	2.875
16	256	33	17	11.66667	9	7.4	6.333333	5.571429	5	4.555556	4.2	3.909091	3.666667	3.461538	3.285714	3.133333	3
17	289	35	18	12.33333	9.5	7.8	6.666667	5.857143	5.25	4.777778	4.4	4.090909	3.833333	3.615385	3.428571	3.266667	3.125
18	324	37	19	13	10	8.2	7	6.142857	5.5	5	4.6	4.272727	4	3.769231	3.571429	3.4	3.25
19	361	39	20	13.66667	10.5	8.6	7.333333	6.428571	5.75	5.222222	4.8	4.454545	4.166667	3.923077	3.714286	3.533333	3.375
20	400	41	21	14.33333	11	9	7.666667	6.714286	6	5.444444	5	4.636364	4.333333	4.076923	3.857143	3.666667	3.5
21	441	43	22	15	11.5	9.4	8	7	6.25	5.666667	5.2	4.818182	4.5	4.230769	4	3.8	3.625
22	484	45	23	15.66667	12	9.8	8.333333	7.285714	6.5	5.888889	5.4	5	4.666667	4.384615	4.142857	3.933333	3.75
23	529	47	24	16.33333	12.5	10.2	8.666667	7.571429	6.75	6.111111	5.6	5.181818	4.833333	4.538462	4.285714	4.066667	3.875
24	576	49	25	17	13	10.6	9	7.857143	7	6.333333	5.8	5.363636	5	4.692308	4.428571	4.2	4
25	625	51	26	17.66667	13.5	11	9.333333	8.142857	7.25	6.555556	6	5.545455	5.166667	4.846154	4.571429	4.333333	4.125
26	676	53	27	18.33333	14	11.4	9.666667	8.428571	7.5	6.777778	6.2	5.727273	5.333333	5	4.714286	4.466667	4.25
27	729	55	28	19	14.5	11.8	10	8.714286	7.75	7	6.4	5.909091	5.5	5.153846	4.857143	4.6	4.375
28	784	57	29	19.66667	15	12.2	10.33333	9	8	7.222222	6.6	6.090909	5.666667	5.307692	5	4.733333	4.5
29	841	59	30	20.33333	15.5	12.6	10.66667	9.285714	8.25	7.444444	6.8	6.272727	5.833333	5.461538	5.142857	4.866667	4.625
30	900	61	31	21	16	13	11	9.571429	8.5	7.666667	7	6.454545	6	5.615385	5.285714	5	4.75
31	961	63	32	21.66667	16.5	13.4	11.33333	9.857143	8.75	7.888889	7.2	6.636364	6.166667	5.769231	5.428571	5.133333	4.875
32	1024	65	33	22.33333	17	13.8	11.66667	10.14286	9	8.111111	7.4	6.818182	6.333333	5.923077	5.571429	5.266667	5
33	1089	67	34	23	17.5	14.2	12	10.42857	9.25	8.333333	7.6	7	6.5	6.076923	5.714286	5.4	5.125
34	1156	69	35	23.66667	18	14.6	12.33333	10.71429	9.5	8.555556	7.8	7.181818	6.666667	6.230769	5.857143	5.533333	5.25
35	1225	71	36	24.33333	18.5	15	12.66667	11	9.75	8.777778	8	7.363636	6.833333	6.384615	6	5.666667	5.375
36	1296	73	37	25	19	15.4	13	11.28571	10	9	8.2	7.545455	7	6.538462	6.142857	5.8	5.5
37	1369	75	38	25.66667	19.5	15.8	13.33333	11.57143	10.25	9.222222	8.4	7.727273	7.166667	6.692308	6.285714	5.933333	5.625
38	1444	77	39	26.33333	20	16.2	13.66667	11.85714	10.5	9.444444	8.6	7.909091	7.333333	6.846154	6.428571	6.066667	5.75
39	1521	79	40	27	20.5	16.6	14	12.14286	10.75	9.666667	8.8	8.090909	7.5	7	6.571429	6.2	5.875
40	1600	81	41	27.66667	21	17	14.33333	12.42857	11	9.888889	9	8.272727	7.666667	7.153846	6.714286	6.333333	6
41	1681	83	42	28.33333	21.5	17.4	14.66667	12.71429	11.25	10.11111	9.2	8.454545	7.833333	7.307692	6.857143	6.466667	6.125
42	1764	85	43	29	22	17.8	15	13	11.5	10.33333	9.4	8.636364	8	7.461538	7	6.6	6.25
43	1849	87	44	29.66667	22.5	18.2	15.33333	13.28571	11.75	10.55556	9.6	8.818182	8.166667	7.615385	7.142857	6.733333	6.375
44	1936	89	45	30.33333	23	18.6	15.66667	13.57143	12	10.77778	9.8	9	8.333333	7.769231	7.285714	6.866667	6.5
45	2025	91	46	31	23.5	19	16	13.85714	12.25	11	10	9.181818	8.5	7.923077			
46	2116	93	47	31.66667	24	19.4	16.33333	14.14286	12.5	11.22222	10.2	9.363636	8.666667	8.076923			

An additional perspective to consider is that in order for squares (*2-powers*) to be Pythagorean - and represent *2-diffs*, the following must also apply (where a=z-x and b=y-x):-

$$x^2+2bx+b^2 = 2ax+a^2,$$

solving for x also results in:

$$x = a-b+\sqrt{(2a^2 - 2ab)}$$

Tabular representations of a, b, and x reveal the relative paucity of integers hidden among the Pythagorean range but constant in terms of absolute ratios of a:b. For example all of the **3-4-5 triads** are found within a:b ratios of ***2:1***. All of the **5-12-13 triads** occur within ratios of ***8:7*** and all of the **8-15-17 triads** within a:b ratios of ***9:7***. Additionally, all of the b=1 triads are found within a:b ratios of: **$2x^2$-1 : $2x^2$.** And that is simply because *when z-y=1*

$$x \to 2x_1+1,\ y \to 2x_1^2 + 2x_1 \text{ and } z \to 2x_1^2 + 2x_1 + 1 \textit{ and}$$

$$a=y-x \to 2x_1^2-1 \text{ and } b=z-x \to 2x_1^2$$

b=y-x	a=z-x	x "=a-b+SQRT(ABS(2*a^2 - 2*a*b))"	x^2+2bx+b^2	2ax+a^2	x^2+2bx+b^2 = 2ax+a^2	"(x)"	"y=(x+b)"	"z=(x+a)"	"=(x+a) - (x+b)"	"=x^2"	"=y^2"	"=z^2"
1	0	#REF!	#REF!	#REF!								
1`	1	"=B3-A3+SQRT(ABS(2*B3^2 - 2*A3*B3))"	#VALUE!	#VALUE!								
1	2	3	16	16	(3-4-5)	3	4	5	1	9	16	25
1	3	5.464101615	41.78460969	41.78460969		5.464101615	6.464101615	8.464101615	2	29.85640646	41.78460969	71.64101615
1	4	7.898979486	79.19183588	79.19183588		7.898979486	8.898979486	11.89897949	3	62.39387691	79.19183588	141.5857128
1	5	10.32455532	128.2455532	128.2455532		10.32455532	11.32455532	15.32455532	4	106.5964426	128.2455532	234.8419958
1	6	12.74596669	188.9516003	188.9516003		12.74596669	13.74596669	18.74596669	5	162.4596669	188.9516003	351.4112672
1	7	15.16515139	261.3121195	261.3121195		15.16515139	16.16515139	22.16515139	6	229.9818167	261.3121195	491.2939361
1	8	17.58300524	345.3280839	345.3280839		17.58300524	18.58300524	25.58300524	7	309.1620734	345.3280839	654.4901573
1	9	20	441	441	(20-21-29)	20	21	29	8	400	441	841
1	10	22.41640786	548.3281573	548.3281573		22.41640786	23.41640786	32.41640786	9	502.4953416	548.3281573	1050.823499
1	11	24.83239697	667.3127334	667.3127334		24.83239697	25.83239697	35.83239697	10	616.6479395	667.3127334	1283.960673
1	12	27.24807681	797.9538434	797.9538434		27.24807681	28.24807681	39.24807681	11	742.4576898	797.9538434	1540.411533
1	13	29.66352173	940.251565	940.251565		29.66352173	30.66352173	42.66352173	12	879.9245216	940.251565	1820.176087
1	14	32.07878403	1094.205953	1094.205953		32.07878403	33.07878403	46.07878403	13	1029.048385	1094.205953	2123.254338
1	15	34.49390153	1259.817046	1259.817046		34.49390153	35.49390153	49.49390153	14	1189.829243	1259.817046	2449.646289
1	16	36.9089023	1437.084874	1437.084874		36.9089023	37.9089023	52.9089023	15	1362.267069	1437.084874	2799.351943
1	17	39.32380758	1626.009458	1626.009458		39.32380758	40.32380758	56.32380758	16	1546.361843	1626.009458	3172.3713
1	18	41.73863375	1826.590815	1826.590815		41.73863375	42.73863375	59.73863375	17	1742.113548	1826.590815	3568.704363
1	19	44.15339366	2038.828959	2038.828959		44.15339366	45.15339366	63.15339366	18	1949.522172	2038.828959	3988.351131
1	20	46.5680975	2262.7239	2262.7239		46.5680975	47.5680975	66.5680975	19	2168.587705	2262.7239	4431.311605
1	21	48.98275349	2498.275647	2498.275647		48.98275349	49.98275349	69.98275349	20	2399.31014	2498.275647	4897.585786
1	22	51.39736831	2745.484206	2745.484206		51.39736831	52.39736831	73.39736831	21	2641.689469	2745.484206	5387.173674
1	23	53.81194744	3004.349582	3004.349582		53.81194744	54.81194744	76.81194744	22	2895.725687	3004.349582	5900.07527
1	24	56.22649545	3274.871782	3274.871782		56.22649545	57.22649545	80.22649545	23	3161.418791	3274.871782	6436.290572
1	25	58.64101615	3557.050808	3557.050808		58.64101615	59.64101615	83.64101615	24	3438.768775	3557.050808	6995.819583
1	26	61.05551275	3850.886663	3850.886663		61.05551275	62.05551275	87.05551275	25	3727.775638	3850.886663	7578.662301
1	27	63.46998799	4156.379351	4156.379351		63.46998799	64.46998799	90.46998799	26	4028.439376	4156.379351	8184.818727
1	28	65.88444419	4473.528875	4473.528875		65.88444419	66.88444419	93.88444419	27	4340.759986	4473.528875	8814.288861
1	29	68.29888336	4802.335235	4802.335235		68.29888336	69.29888336	97.29888336	28	4664.737468	4802.335235	9467.072703
1	30	70.71330723	5142.798434	5142.798434		70.71330723	71.71330723	100.7133072	29	5000.371819	5142.798434	10143.17025
1	31	73.12771731	5494.918473	5494.918473		73.12771731	74.12771731	104.1277173	30	5347.663038	5494.918473	10842.58151
1	32	75.5421149	5858.695354	5858.695354		75.5421149	76.5421149	107.5421149	31	5706.611124	5858.695354	11565.30648
1	33	77.95650117	6234.129077	6234.129077		77.95650117	78.95650117	110.9565012	32	6077.216075	6234.129077	12311.34515
1	34	80.37087713	6621.219645	6621.219645		80.37087713	81.37087713	114.3708771	33	6459.477891	6621.219645	13080.69754
1	35	82.78524367	7019.967057	7019.967057		82.78524367	83.78524367	117.7852437	34	6853.39657	7019.967057	13873.36363
1	36	85.19960159	7430.371315	7430.371315		85.19960159	86.19960159	121.1996016	35	7258.972111	7430.371315	14689.34343
1	37	87.6139516	7852.432419	7852.432419		87.6139516	88.6139516	124.6139516	36	7676.204515	7852.432419	15528.63693
1	38	90.02829433	8286.150369	8286.150369		90.02829433	91.02829433	128.0282943	37	8105.093781	8286.150369	16391.24415
1	39	92.44263036	8731.525168	8731.525168		92.44263036	93.44263036	131.4426304	38	8545.639907	8731.525168	17277.16507
1	40	94.85696018	9188.556814	9188.556814		94.85696018	95.85696018	134.8569602	39	8997.842894	9188.556814	18186.39971
1	41	97.27128425	9657.245309	9657.245309		97.27128425	98.27128425	138.2712843	40	9461.70274	9657.245309	19118.94805
1	42	99.685603	10137.59065	10137.59065		99.685603	100.685603	141.685603	41	9937.219446	10137.59065	20074.8101
1	43	102.0999168	10629.59285	10629.59285		102.0999168	103.0999168	145.0999168	42	10424.39301	10629.59285	21053.98586
1	44	104.514226	11133.25189	11133.25189		104.514226	105.514226	148.514226	43	10923.22944	11133.25189	22056.47532
1	45	106.9285309	11648.56778	11648.56778		106.9285309	107.9285309	151.9285309	44	11433.71072	11648.56778	23082.2785
1	46	109.3428318	12175.54052	12175.54052		109.3428318	110.3428318	155.3428318	45	11955.85486	12175.54052	24131.39538
1	47	111.7571289	12714.17012	12714.17012		111.7571289	112.7571289	158.7571289	46	12489.65586	12714.17012	25203.82597
1	48	114.1714225	13264.45656	13264.45656		114.1714225	115.1714225	162.1714225	47	13035.11371	13264.45656	26299.57027
1	49	116.5857128	13826.39985	13826.39985		116.5857128	117.5857128	165.5857128	48	13592.22843	13826.39985	27418.62828
1	50	119	14400	14400	(119-120-169)	119	120	169	49	14161	14400	28561

										0	0	0
7	7	0	49	49		0	7	7	0	0	49	49
7	8	5	144	144	{5-12-13}	5	12	13	1	25	144	169
7	9	8	225	225	{8-15-17}	8	15	17	2	64	225	289
7	10	10.74596669	314.9193338	314.9193338		10.74596669	17.74596669	20.74596669	3	115.4758002	314.9193338	430.395134
7	11	13.38083152	415.3782934	415.3782934		13.38083152	20.38083152	24.38083152	4	179.0466522	415.3782934	594.4249456
7	12	15.95445115	526.9068276	526.9068276		15.95445115	22.95445115	27.95445115	5	254.5445115	526.9068276	781.4513391
7	13	18.489996	649.7398959	649.7398959		18.489996	25.489996	31.489996	6	341.879952	649.7398959	991.6198479
7	14	21	784	784	{3-4-5}	21	28	35	7	441	784	1225
7	15	23.49193338	929.7580015	929.7580015		23.49193338	30.49193338	38.49193338	8	551.8709342	929.7580015	1481.628936
7	16	25.97056275	1087.058008	1087.058008		25.97056275	32.97056275	41.97056275	9	674.4701295	1087.058008	1761.528137
7	17	28.43908891	1255.929023	1255.929023		28.43908891	35.43908891	45.43908891	10	808.7817783	1255.929023	2064.710801
7	18	30.89974874	1436.390955	1436.390955		30.89974874	37.89974874	48.89974874	11	954.7944723	1436.390955	2391.185427
7	19	33.3541565	1628.457947	1628.457947		33.3541565	40.3541565	52.3541565	12	1112.499756	1628.457947	2740.957703
7	20	35.8035085	1832.14034	1832.14034		35.8035085	42.8035085	55.8035085	13	1281.891221	1832.14034	3114.031561
7	21	38.24871131	2047.445875	2047.445875		38.24871131	45.24871131	59.24871131	14	1462.963917	2047.445875	3510.409791
7	22	40.69046516	2274.380467	2274.380467		40.69046516	47.69046516	62.69046516	15	1655.713955	2274.380467	3930.094422
7	63	140	21609	21609	{20-21-29}	140	147	203	56	19600	21609	41209
7	128	297	92416	92416	{297-304-425}	297	304	425	121	88209	92416	180625
17	17	0	289	289		0	17	17	0	0	289	289
17	18	7	576	576	{7-24-25}	7	24	25	1	49	576	625
17	19	10.71779789	768.2763197	768.2763197		10.71779789	27.71779789	29.71779789	2	114.8711915	768.2763197	883.1475113
17	20	13.95445115	958.178046	958.178046		13.95445115	30.95445115	33.95445115	3	194.7267069	958.178046	1152.904753
17	21	16.9614814	1153.382219	1153.382219		16.9614814	33.9614814	37.9614814	4	287.6918512	1153.382219	1441.07407
17	22	19.83239697	1356.625467	1356.625467		19.83239697	36.83239697	41.83239697	5	393.3239697	1356.625467	1749.949437
17	23	22.61324773	1569.209395	1569.209395		22.61324773	39.61324773	45.61324773	6	511.3589727	1569.209395	2080.568368
17	24	25.33030278	1791.854533	1791.854533		25.33030278	42.33030278	49.33030278	7	641.6242389	1791.854533	2433.478772
17	25	28	2025	2025	{28-45-53}	28	45	53	8	784	2025	2809
17	26	30.63330765	2268.931998	2268.931998		30.63330765	47.63330765	56.63330765	9	938.3995378	2268.931998	3207.331536
17	27	33.23790008	2523.846604	2523.846604		33.23790008	50.23790008	60.23790008	10	1104.758002	2523.846604	3628.604606
17	28	35.81934729	2789.883448	2789.883448		35.81934729	52.81934729	63.81934729	11	1283.02564	2789.883448	4072.909089
17	29	38.38181192	3067.145091	3067.145091		38.38181192	55.38181192	67.38181192	12	1473.163486	3067.145091	4540.308577
17	30	40.92848009	3355.708805	3355.708805		40.92848009	57.92848009	70.92848009	13	1675.140482	3355.708805	5030.849288
17	31	43.46183973	3655.634063	3655.634063		43.46183973	60.46183973	74.46183973	14	1888.951512	3655.634063	5544.565575
17	32	45.98386677	3966.967473	3966.967473		45.98386677	62.98386677	77.98386677	15	2114.516003	3966.967473	6081.483476
17	33	48.49615362	4289.746139	4289.746139		48.49615362	65.49615362	81.49615362	16	2351.876916	4289.746139	6641.623055
17	34	51	4624	4624	{3-4-5}	51	68	85	17	2601	4624	7225
17	35	53.4964787	4969.753509	4969.753509		53.4964787	70.4964787	88.4964787	18	2861.873233	4969.753509	7831.626742
17	26	30.63330765	2268.931998	2268.931998		30.63330765	47.63330765	56.63330765	9	938.3995378	2268.931998	3207.331536
17	37	58.47076812	5695.836841	5695.836841		58.47076812	75.47076812	95.47076812	20	3418.830725	5695.836841	9114.667566
17	38	60.94996871	6076.197622	6076.197622		60.94996871	77.94996871	98.94996871	21	3714.898686	6076.197622	9791.096308
17	39	63.42463035	6468.121168	6468.121168		63.42463035	80.42463035	102.4246304	22	4022.683736	6468.121168	10490.8049
17	40	65.89522118	6871.617694	6871.617694		65.89522118	82.89522118	105.8952212	23	4342.180174	6871.617694	11213.79787
17	41	68.36214603	7286.695974	7286.695974		68.36214603	85.36214603	109.362146	24	4673.383009	7286.695974	11960.07898
17	42	70.82575695	7713.363584	7713.363584		70.82575695	87.82575695	112.8257569	25	5016.287847	7713.363584	12729.65143
17	43	73.28636167	8151.627104	8151.627104		73.28636167	90.28636167	116.2863617	26	5370.890807	8151.627104	13522.51791
17	44	75.74423043	8601.492278	8601.492278		75.74423043	92.74423043	119.7442304	27	5737.188443	8601.492278	14338.68072
17	45	78.19960159	9062.964143	9062.964143		78.19960159	95.19960159	123.1996016	28	6115.177689	9062.964143	15178.14183
17	46	80.65268628	9536.047138	9536.047138		80.65268628	97.65268628	126.6526863	29	6504.855804	9536.047138	16040.90294
17	47	83.10367219	10020.74519	10020.74519		83.10367219	100.1036722	130.1036722	30	6906.220331	10020.74519	16926.96552
17	48	85.55272679	10517.06177	10517.06177		85.55272679	102.5527268	133.5527268	31	7319.269061	10517.06177	17836.33083
17	49	88	11025	11025	{88-105-137}	88	105	137	32	7744	11025	18769
17	98	207	50176	50176	{207-224-305}	207	224	305	81	42849	50176	93025
19	38	57	5776	5776	{3-4-5}	57	76	95	19	3249	5776	9025
31	32	9	1600	1600	{9-40-41}	9	40	41	1	81	1600	1681
										0	0	0
49	50	11	3600	3600	{11-60-61}	11	60	61	1	121	3600	3721
										0	0	0
71	72	13	7056	7056	{13-84-85}	13	84	85	1	169	7056	7225
79	81	20	9801	9801	{20-99-101}	20	99	101	2	400	9801	10201
359	361	40	159201	159201	{40-399-401}	40	399	401	2	1600	159201	160801
1087	1089	68	1334025	1334025	{68-1155-1157}	68	1155	1157	2	4624	1334025	1338649
2599	2601	104	7306209	7306209	{104-2703-2705}	104	2703	2705	2	10816	7306209	7317025

Another way of looking at this is to consider a **Fundamental Theorem of Exponentiation** as the element identified as **#6** below **(*<u>which arises in support of FLT</u>*.)**

1) **Let $r = b/a$, then $r^n = b^n/a^n$, $b = ar$ and $b^n = a^n r^n$**
2) **If $x^n + a^n = b^n$, then**
   a. **$x^n/(b-a) = (b^n - a^n)/(b-a)$**
   b. **$= (b^{n-1} + ab^{n-2} + \ldots + a^{n-2}b + a^{n-1})$**
   c. **$= (b - \zeta_{n.1}a)(b - \zeta_{n.2}a) \ldots (b - \zeta_{n.n-2}a)(b - \zeta_{n.n-1}a)$**
3) **$x^n/(b-a)^n = (b^n - a^n)/(b-a)^{n-1}$**
4) **$= (a^n r^n - a^n)/(ar-a)^n$**
5) **$= (a^n)(r^n - 1)/(a^n)(r - 1)^n$**

6) **$= (r^n - 1)/(r - 1)^n$ (which *cannot* be an n^{th} power >2 unless r=1 or 0)***
$\neq x^n/(b-a)^n$ (which *is* an n^{th} power) if $x,a,b \in Q \neq 0$ or $b \neq a$ and $n>2$

***In the preceding table of r^5 note how values approach 1 as a^5 increases (Column 2) for all values of b^5 (rows increasing horizontally and vertically):-**

Ergo *COROLLARY #1*

if $b = a + c$, then $x^n = (b^n - a^n) = (a + c)^n - a^n$ and $a^{n-1} + ca^{n-2} + \ldots + c^{n-1}a + c^n \neq x^n$ if $x,a,b,c \in Q \neq 0$ and $n>2$

II. Sequelae

1) Let's define an ***algebraic integer*** **(ζ)** as a polynomial in one variable (*x*) of degree ***n*** with the n-th degree term possessing a coefficient =1, or: -

$$\zeta = x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$$

2) Let's re-formulate the same algebraic integer **(ι)** in terms of *base* **x** replacing the "+" sign with a dot, "·" and using the *base* **x** as a terminal $_{\text{subscript}}$ or:-

$$\zeta_x = (1 \cdot a_1 \cdot a_2 \cdot \ldots \cdot a_{n-2} \cdot a_{n-1} \cdot a_n)_x$$

3) If one forms a **universal power function** for $\iota = (x+a)^n$ by replacing the $_{\text{subscripts}}$ for **a** as $^{\text{superscripts}}$, then one could just as easily write: -

$$\zeta_a = 1 \cdot x \cdot x^2 \cdot \ldots \cdot x^{n-2} \cdot x^{n-1} \cdot x^n{}_a \text{ or } \zeta_x = 1 \cdot a \cdot a^2 \cdot \ldots \cdot a^{n-2} \cdot a^{n-1} \cdot a^n{}_x)$$

4) Note that the *constant* term here, x^n or a^n, or "χ" of the algebraic integer **(ζ)** and although dependent upon the base variable itself (**x**) [or (**a**)] remains independent of the value taken for that base variable. (Sound confusing? If not, you are obviously not trying hard enough. ☺)

5) Lemma I: - It follows directly from **(4)** that if an algebraic integer **(ζ)** is a **universal power function** of some variable**, x,** or variables**, (x + a)** - i.e., square, cube, 4th, 5th, nth, then its constant, χ_x also represents some integer ($\zeta \ni \mathbb{Z}$) to that same power. It also follows as the night the day that: -

$$\chi_x = \zeta \bmod(x) = a^n \text{ (and } \chi_a = \zeta \bmod(a) = x^n.)$$

6) Consider two right-triangles, **A** and **B** (*Figure 1*) which are related in that

a) the base of the larger triangle (**B**) is the square of the base of the smaller triangle (**A**)

b) the difference between the hypotenuse and the vertical arms of both triangles is the same and equal to some measure, "**a**"

c) we will be interested in framing the vertical arms, "***x***" and "***y***", of the both triangles, (**A**) and (**B**), as functions of the vertical arm, "***d"***, of the smaller triangle, (**A**), as well as of the difference, "a".

$$x = (\omega = x^{1/2})^2 = (d + a)^2 - d^2 = (2ad + a^2) = \underline{2a \cdot a^2{}_d}$$

$$x^2 = (y + a)^2 - y^2 = (2ay + a^2) = (2ad + a^2)^2 = 4a^2d^2 + 4a^3d + \underline{a^4}$$

$$2ay = 4a^2d^2 + 4a^3d + a^4 - a^2$$

$$y = 2ad^2 + 2a^2d + (a^3 - a)/2 = \underline{2a \cdot 2a^2 \cdot (a^3 - a)/2_d}$$

$$z = y + a = \underline{2a \cdot 2a^2 \cdot (a^3 + a)/2_d}$$

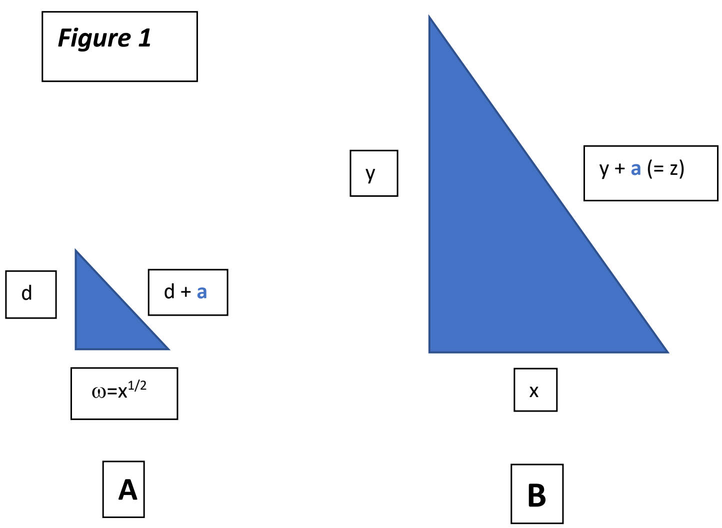


Figure 1

Commented [G11]:

7) Note that when checking whether $y^2 + x^2 = z^2$: -

$$4a^2 \cdot 8a^3 \cdot 6a^4 - 2a^2 \cdot 2(a^5 - a^3) \cdot (a^6 - 2a^4 + a^2)/4_d$$

$$+\ 0 \cdot 0 \cdot \quad 4a^2 \cdot \quad 4a^3 \cdot \quad (4a^4) \quad /4_d$$

$$= 4a^2 \cdot 8a^3 \cdot 6a^4 + 2a^2 \cdot 2(a^5 + a^3) \cdot (a^6 + 2a^4 + a^2)/4_d,$$

also that when a =1 this reduces to: - $2\cdot1^2{}_d + 2\cdot2\cdot0^2{}_d = 2\cdot2\cdot1^2{}_d$,

adding the additional restriction of d = 1, *then x=3, y=4, and z=5*)

8) Why do we do this and what are we, in fact doing to accomplish this?

a) The variable **x** as defined becomes an obvious linear function of **d** (and a quadratic function of **a)** in (**A**).

b) Additionally, we have defined "**x**" as some square, "ω^2", which is, itself, not only a difference in squares (**A**), but also a difference in squares when squared (**B**) tying (**A**) to (**B**) in a way such that the fixed difference between the other two variables **(y + a) - y, (d + a) - d**, is the same, "**a**".

c) In that process, we have made the vertical variable "**y**" (and "**z**") also a quadratic function of **d** (as well as a 3rd degree function of **a**.)

d) The variable "**d**" then becomes a base or basis from which all Pythagorean triangles/triples can be defined, catalogued, and understood.

e) One can also grasp intuitively from these relations that **x**, **y**, and **z** have infinite elements in all fields, but that this is especially obvious in **Z**.

9) As in **Lemma 1**, consider just the χ values of the equations in (**7**): -

$$(a^6 - 2a^4 + a^2)/4_d + (4a^4)/4_d = (a^6 + 2a^4 + a^2)/4_d$$

And note how all the χ values are, indeed, perfect squares themselves, as are the χ values of the χ values (!). Note the internal consistency of the χ values in **Table I**. The χ values of the base_{10} representations are also consistent: -

e.g., (for a=3, $12^2 + 9^2 =$

$15^2 \Leftarrow 144 + 81 \Rightarrow 225)_{10}$

Table I

a	d	$(a^6 - 2a^4 + a^2)/4_d$	$+ (4a^4)/4_d$	$= (a^6 + 2a^4 + a^2)/4_d$
0	1	0^2	0^2	0^2
1	1	0^2	1^2	1^2
2	1	3^2	4^2	5^2
3	1	12^2	9^2	15^2
4	1	30^2	16^2	34^2

10) Observe the level of complexity that this analysis suggests for just a putatively simple 2nd degree elliptical equation. If we apply the same logic to one of the third degree, i.e., : - $x^3 + y^3 = z^3$

then we arrive at the following: -

$$x = (d + a)^3 – d^3 = 3ad^2 + 3a^2d + a^3$$

$$x^3 = (y + a)^3 – y^3 = 3ay^2 + 3a^2y + a^3$$

$$x^3 = 3ay^2 + 3a^2y + a^3 = (3ad^2 + 3a^2d + a^3)^3$$

$$= 27a^3d^6 + 81a^4d^5 +108a^5d^4 + 81a^6d^3 + 36a^7d^2 + 9a^8d + \underline{a^9}$$

$$y^2 + ay = 9a^2d^6 + 27a^3d^5 +36a^4d^4 + 27a^5d^3 + 12a^6d^2 + 3a^7d + (a^2 – a^8)/3$$

$$y = (-a +/- (36a^2d^6 + 108a^3d^5 +144a^4d^4 + 108a^5d^3 + 48a^6d^2 + 12a^7d + (4a^8 – a^2)/3)^{1/2})/2$$

$$= (-a +/- (36a^2 \cdot 108a^3 : 144a^4 \cdot 108a^5 \cdot 48a^6 \cdot 12a^7 \cdot (4a^8 – a^2)/3)^{1/2}{}_d/2$$

Here if a,d = 1, then x=7, y ~ 10.188779, z~11.188779, note that x ϶ $\mathbb{Z}$, but y,z ϶ $\mathbb{R}$.

Checking, [343 + 1057.709604 ~= 1400.709603]

If the general difference representation (f) of the cubic difference $(x+a)^3 - x^3$ is, indeed,

$$\mathbf{3ax^2+3a^2x+a^3}$$

Then,

$$f(y)=3ay^2+3a^2y+a^3 =(3ax^2+3a^2x+a^3)^3$$

and

$$f(y)= (27a^3+81a^4+108a^5+81a^6+36a^7+9a^8+a^9{}_x)$$

and

$$f(y)/a= (27a^2+81a^3+108a^4+81a^5+36a^6+9a^7+a^8{}_x)= 3y^2+3ay+a^2$$

and

$$3y^2+3ay+a^2 - (27a^2+81a^3+108a^4+81a^5+36a^6+9a^7+a^8{}_x) =0$$

and

Input interpretation

solve	$3y^2+3ay-(27x^6a^2+81x^5a^3+108x^4a^4+81x^3a^5+36x^2a^6+9xa^7+a^2-a^8)=0$	for	y

Results

$$y=\frac{1}{6}\left(-\sqrt{3}\sqrt{(-4a^8+36a^7x+144a^6x^2+324a^5x^3+432a^4x^4+324a^3x^5+108a^2x^6+7a^2)}-3a\right)$$

$$y=\frac{1}{6}\left(\sqrt{3}\sqrt{(-4a^8+36a^7x+144a^6x^2+324a^5x^3+432a^4x^4+324a^3x^5+108a^2x^6+7a^2)}-3a\right)$$

11) If we generalize to: - $x^n + y^n = z^n$, and consider $x^n = (y + a)^n - y^n$, it follows easily from **7)**, **9)** and **10)** that $\chi(x^n)_d = (a^n)^n)$ [$=a^{n\text{-squared}}$]. The question arises, "Does $a^{n\text{-squared}} \mid x^n$, (i.e. evenly divide x^n or is it a factor of x^n), and, if so, when?"

12) If $x \ni \mathbb{Z}$, then $x^n \ni \mathbb{Z}$, and so must $(y + a)^n - y^n \ni \mathbb{Z}$. Then if $y \ni \mathbb{Z}$, so must z, y^n, $z^n \ni \mathbb{Z}$.

13) When **n=2**, then x^2/ a^4 becomes $((y + a)^2 - y^2)^2/ a^4$ or: -

$(4a^2d^2 + 4a^3d + \underline{a^4})/a^4 \Rightarrow \boldsymbol{1 \cdot 4d \cdot 4d^2 \cdot 0 \cdot 0_a / 1 \cdot 0 \cdot 0 \cdot 0 \cdot 0_a}$

$= \boldsymbol{1\,\underline{.}\,4d \cdot 4d^2_a}$ (note the "$\underline{a}^{th}$"-imal point) $\underline{\boldsymbol{= 9}}$ or $\mathbf{3^2}$ when $a = d$!

14) When **n=3**, then x^3/ a^9 becomes $((y + a)^3 - y^3)^3/ a^9$ or: -

$(27a^3d^6 + 81a^4d^5 + 108a^5d^4 + 81a^6d^3 + 36a^7d^2 + 9a^8d + \underline{a^9})/a^9$

$\Rightarrow \boldsymbol{1 \cdot 9d \cdot 36d^2 \cdot 81d^3 \cdot 108d^4 \cdot 81d^5 \cdot 27d^6 \cdot 0 \cdot 0 \cdot 0a / 1 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0_a}$

$= \boldsymbol{1\,\underline{.}\,9d \cdot 36d^2 \cdot 81d^3 \cdot 108d^4 \cdot 81d^5 \cdot 27d^6 \cdot 0 \cdot 0 \cdot 0_a}$ (note the "$\underline{a}^{th}$"-imal point)

$\underline{\boldsymbol{= 343}}$ or $\mathbf{7^3}$ when $a = d$!

15) Lemma 2: - $x^n/ a^{n\text{-squared}} = (2^n - 1)^n$ when $a = d$!

16) When **n=2**, then x^2/ a^4 also becomes $((y + a)^2 - y^2)/ a^4$ or:

$(2ay + a^2) / a^4 \Rightarrow \mathbf{1 \cdot 2y \cdot 0\,_a / 1 \cdot 0 \cdot 0 \cdot 0 \cdot 0\,_a = 0\,.\,0 \cdot 0 \cdot 1 \cdot 2y\,_a}$

From **(7)** we know that y=4 when a=d=1, then also in that same case,

$(2ay + a^2) / a^4 = (8 + 1) /1$ $\underline{\boldsymbol{= 9}}$ or $\mathbf{3^2}$

16) When **n=3**, then x^3/ a^9 also becomes $((y + a)^3 - y^3)/ a^9$ or: -

$3ay^2 + 3a^2y + a^3 / a^9 \Rightarrow \boldsymbol{1 \cdot 3y \cdot 3y^2 \cdot 0a / 1 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0_a}$

$\mathbf{= 0.\,0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 1 \cdot 3y \cdot 3y^2{}_a}$

From **(10)** we know that y ~ 10.188779 when a=1, (and d=1) then also in that same case, $3ay^2 + 3a^2y + a^3 / a^9$ ~ =342.999989 ~= ***343*** or 7^3**17).** For every k **ε Q** and x **ε Z,** since k(2x+1) includes every integer, then I submit that ***every integral power n >2 of that integer can be expressed as a difference between squares* - while also representing the differences between the integers themselves**

================================||==================================

square+	cube=	square	||	square+	quartic=	square
0	0	0	||	0	0	0
0	1	1	||	0	1	1
1	2	3	||	3	2	5
3	3	6	||	12	3	15
6	4	10	||	30	4	34
10	5	15	||	60	5	65
x(x-1)/2			||	x(x-1)(x+1)/2		

================================||==================================

square+	quintic=	square	||	square+	heptic=	square
0	0	0	||	0	0	0
0	1	1	||	0	1	1
7 `	2	9	||	15	2	17
39	3	42	||	120	3	123
126	4	130	||	510	4	514
310	5	315	||	1560	5	1565
$x(x-1)(x^2+x+1)/2$			||	$x(x-1)(x^3+x^2+x+1)/2$		

where the initial terms are represented by $((x^{n-1}-x))/2$

so $((x)((x^{n-2}+x))/2) - ((x)((x^{n-2}-1))/2)^2 = x^n$

In one sense this does show a function representing the differences among powers, i.e., :- If

$f_1((x+1)^{n-2}) = ((x)((x^{n-2}-1))/2 + x)^2$,

$f_z(x^n) = x^n$, and

$f_3((x+1)^{n-2}) = ((x)((x^{n-2}-1))/2)^2$

Then $f_1 - f_2 = f_3$ (for unit integral differences.)

i) In an important sense, this relationship adds a profoundly ironic and yet strong group theoretical perspective to the entire FLT story. For the fundamental two properties here described:

a) $x + y = z$

and

b) $x^2 + y^n = z^2$

to ***hold*** for x,y,z and n ε **Z** note that it is very obvious that ***n must be greater than two*** (2), *i.e.*, it cannot hold for n=2 but does hold for higher values in direct contradistinction to FLT which only holds for powers less than or equal to two (2)!

To restate the obvious, the sum of any two integers can never equal the sum of their ***squares*** – ***except*** when at least one of the values is 0 (zero).

Note that this holds true (as would be expected) for *negativ*e values as well.

square+	cube=	square	square+	quartic=	square
0	0	0	0	0	0
1	-1	0	0	-1	1
3	-2	1	5	-2	3
6	-3	3	15	-3	12
10	-4	6	34	-4	30
15	-5	10	65	-5	60
$x(x-1)/2$			$x(x-1)(x+1)/2$		

square+	quintic=	square	square+	heptic=	square
0	0	0	0	0	0
1	-1	0	1	-1	0
9	-2	7	17	-2	15
42	-3	39	123	-3	120
130	-4	126	514	-4	510
315	-5	310	1565	-5	1560
$x(x-1)(x^2+x+1)/2$			$x(x-1)(x^3+x^2+x+1)/2$		

ii) Note how the $(x-a)(x^n-a^n)/(x-a)^2 = x^{n-1}+ax^{n-2}+...+a^{n-2}x+a^n$ [incorporating the non-trivial roots of unity]family theme keeps recurring here

iii) Note also that since each sequence is unique for each degree, the only differences between rows that are included within each family are differences from that row to the zero line, and, so, where $\mathbf{a^2 + b^n = c^2}$ and $\mathbf{d^2 + e^n = f^2}$, then,

$\mathbf{b^n = c^2 - a^2}$ and $\mathbf{e^n = f^2 - d^2}$, and ***all power differences of integers for n>2 may now be additionally expressed as differences between the sums or differences of squares***:-

$$\mathbf{e^n - b^n = (f^2 - d^2) - (c^2 - a^2)} \text{ or}$$

$$\mathbf{e^n - b^n = (a^2 + f^2) - (c^2 + d^2)}$$

since each difference value or row appears uniquely within this formulation) for every power >2 (except for those differences from the zero row then (*as further support for FLT*) either

$$\mathbf{a^2 = -d^2 \text{ which } \Leftrightarrow a \text{ or } d \ni \mathfrak{C}, \text{ or } a=d=0}$$

(The question further arises as to whether some integer ***g*** exists such that

$$\mathbf{g^n = e^n - b^n = ((a^2 + f^2)^{1/2})^2 - ((c^2 + d^2)^{1/2})^2}$$

$$\mathbf{g = (e^n - b^n)^{1/n} \text{ but}}$$

$$\mathbf{(c^2 - a^2)^{1/2} \textit{ or } (c^2 + d^2)^{1/2} = (g+1)((g+1)^{n-2}-1)/2}$$

all while $\mathbf{e - b = (a + f) - (c + d)}$)

(*Vielleicht* – "**Why is it asking how n>2 does cause the problem rather than discovering how n=2 uniquely differently does not?**")

Also of note are the following related observations:

$$(x^n+x^{-n})^2 - (x^n-x^{-n})^2 = 4$$

and

if ר = $(x^n-y^n)/(x-y)^n$ (n=odd)

max(ר$^{-1}$) = $4^{((n-1)/2)}$

18) By way of another digression, the Pascal triangle can be considered to be composed of elements consisting of how to increase the square, cube, etc by a single unit in each dimensioned “direction” and then moving up to the next degree (by multiplying by 11).

For example, to move from 1 square to four squares, I would need ***2x + 1*** or **3** (of x=1) more squares (one on each of two sides, plus one more to fill the open corner) and to move from four squares to nine squares I would need ***2x + 1*** or **5** (x=2) more squares (two on each of two sides and one more to fill the empty corner. This is represented by $\mathbf{(x+1)^2-x^2 = 2x+1}$. Similarly, to move from one cube to a symmetrical cube of eight cubes I would need $\boldsymbol{3x^2+3x+1}$ or 7 more cubes: ***2x+1*** or 3 more cubes to fill the bottom layer of 2x2 cubes, and 4 more cubes to add to the top layer to complete the eight cubes. Similarly to go to a 3 x 3 cube from a 2x2 cube I need 19 more cubes (5 cubes (2x+1) each (times two or $2x^2+x$) to fill out the two bottom levels plus a full 9 ($(x+1)^2$) more to complete the top level, and this is simply because $(x+1)^3- x^3 = 3x^2+3x+1$ which is $(2x^2+x) + (x^2+2x+1)$ and which when added to the original 2^3 or 8 cubes yields the next symmetrical cube consisting of 27 cubes. But, simply put, every entry into the next higher dimension requires a multiplication by $\mathbf{11_x}$ (i.e., by **x + 1**).

1 1

1 0 x10

2 1 ---> 2 1 x10

Δ=(2 1)

1 0 0 x10

1 2 1 ---> 1 2 1 x10

2 1 0 x10

Δ= (3 3 1)

1 0 0 0 x10

1 3 3 1 ---> 1 3 3 1 x10

1 2 1 0 x10

2 1 0 0 x10

Δ= (4 6 4 1)

1 0 0 0 0 x10

1 4 6 4 1 ----> 1 4 6 4 1

1 3 3 1 0 x10

1 2 1 0 0 x10

2 1 0 0 0 x10

Δ= (5 10 10 5 1)

1 0 0 0 0 0 x10

1 5 10 10 5 1 ---> & etc.

III. Families of the Algebraic Integers (with Parents, Siblings, and Cousins)

1) It is well-established that although scalar multiplication of algebraic integers does not change the roots, it does, in fact, change the ***range*** of the implied function via scaled symmetry as well as changing the discriminant function.
2) What is of interest in this section are groups of integers which, although *changing* the algebraic integer roots, do ***NOT***, in fact, change the ***range*** of the implied function (via simple parallel translational symmetry) nor do they change the *same* discriminant function.
3) One of these transformations, call it f, changes the domain (or base) of an algebraic integer from $x \Rightarrow x+\alpha$
   a) Consider, for example, as a *progenitor*, the cubic difference function

   $$f(x)= 3x^2 +3x + 1$$

   which f takes $X+\alpha$ to ➔

   $$3x^2 + 3x(1+2\alpha) + 3\alpha^2 + 3\alpha + 1$$

   and the Constant = $f(\alpha)$

   i.e., a ***family*** of algebraic integers *which are all identical.*

   Ergo. f *represents an isomorphic* transformation.
   b) Examples of that family include:

$3x^2 +3x + 1$	$(\alpha=0)$
$3x^2 + 9x + 7$	$(\alpha=1)$
$3x^2 + 15x + 19$	$(\alpha=2)$
$3x^2 + 21x + 37$	$(\alpha=3)$

Note in this isomorphism how the constant in the right-most column still mirrors the progenitor (which in this case is second degree), the middle column in this case is of first degree, and the left-most column is constant – (another type of symmetric reflection.)

Note also how as α increases by unity, **the constants** represent the sum of the immediately preceding coefficients, and **are**, indeed, thereby, **the unique elements of the range.**

Moreover, each and every terms in any family are related by the respective subsequential partial differentials of any parent term, i.e., :-

Take any algebraic integer, ζ, in Delinferni format:-

$$\zeta(x) = C_n \cdot C_{n-1} \cdot C_{n-2} \cdot C_{n-3} \cdot \ldots \cdot C_3 \cdot C_2 \cdot C_1 \cdot C_{0x}$$

Then any C_i of the family δ levels below [$\zeta(x+\delta)$] can be expressed by the following partial derivatives with respect to δ:-

$$C_0 = \zeta(\delta-1)/0! \; [= C_n+C_{n-1}+C_{n-2}+C_{n-3}+\ldots+C_3+C_2+C_1+C_0 \text{ when } \delta=1]$$

$$C_1 = \zeta'(\delta-1)/1! \; [= (d\zeta(\delta-1)/d\delta)/1]$$

$$C_2 = \zeta''(\delta-1)/2! \; [= (d\zeta'(\delta-1)/d\delta)/2]$$

$$C_3 = \zeta'''(\delta-1)/3! \; [= (d\zeta''(\delta-1)/d\delta)/6]$$

…

$$C_n = [\zeta^{n\prime}(\delta-1)/n! = (d\zeta^{(n-1)\prime}(\delta-1)/d\delta)/n!]$$

For example take the family of the *5-power*:

$$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

$$1 \cdot 5 \cdot 10 \cdot 10 \cdot 5 \cdot 1_x$$

$1 \cdot 10 \cdot 40 \cdot 80 \cdot 80 \cdot 32_x$

$1 \cdot 15 \cdot 90 \cdot 270 \cdot 405 \cdot 243_{x\ \&etc}$

@ $\delta=1$

$(\delta^5 + 5\,\delta^4 + 10\,\delta^3 + 10\,\delta^2 + 5\,\delta + 1)/1 = 32$

$(5\,\delta^4 + 20\,\delta^3 + 30\,\delta^2 + 20\,\delta + 5)/1 = 80$

$(20\,\delta^3 + 60\,\delta^2 + 60\,\delta + 20)/2 = 80$

$(60\,\delta^2 + 120\,\delta + 60)/6 = 40$

$(120\,\delta + 120)/24 = 10$

$(120)/120 = 1$

There is also a backwards transformation which is also of interest e.g., : -

$3x^2 - 21x + 37$ $(\alpha=-4)$

$3x^2 - 15x + 19$ $(\alpha=-3)$

$3x^2 - 9x + 7$ $(\alpha=-2)$

$3x^2 - 3x + 1$ $(\alpha=-1)$

<===symmetry point===>

$3x^2 + 3x + 1$ $(\alpha=0)$

$3x^2 + 9x + 7$ $(\alpha=1)$

$3x^2 + 15x + 19$ $(\alpha=2)$

$3x^2 + 21x + 37$ $(\alpha=3)$

Note that the symmetry point differs from the progenitor, i.e., there is a slight asymmetry which exists in this familial representation. Note also that the discriminant, **b^2-4ac**, is the same throughout, i.e., **3.**

4) We can also further extend this rather "close-knit" family to a super-family by using α once again this time to take

$$3\,ax^2 + 3a^2\,x + a^3$$

to ➔

$$3ax^2 + 3x(2a\,\alpha + a^2) + 3a\alpha^2 + 3a^2\alpha + a^3$$

(and where the same family constraints apply as before – note how the format of the constant remails unchanged (or how, as it were, "*phylogeny recapitulates ontogeny"*)

5) The *Universal Difference Families* , when mirrored, change from $(x+a)^n - x^n$ to $(ax)^n - a^n)$.

6) The *Universal Super-Power Families* themselves reduce to (in the case of n=3) $x^3 + ax^2 + 3x(2a\,\alpha+a^2) + 3a\alpha^2 + 3a^2\alpha + a^3$

7) It is in this sense that **FLT** can be reduced to :
What is the intersection of the familial rational ***ranges*** of universal representations of $[x^n]$ and $[(x+a)^n - x^n]$ when n>2,

and x, a, are all non-zero rational?

a) **All *families* have the same discriminant. Power *family* discriminants are all *0*. The absolute value of universal n^th^ power *a*-difference *family* discriminants is $a^{(n^2-n-2)}n^{n-2}$.**
   N.B. *They are only equal when a=0.*

b) **Universal n^{th} power *families* have n+1 digits.**

c) **The *family* of x^n is the same as $(x+a)^n$. All digitally expressed universal powers have universal *mirrors* $(a+x)^n$.**

d) **The constant of the *family* $(x+a)^n$ is a^n.**

e) **Universal n^{th} power difference *families* have only n digits**

f) **When *mirrored* (assuming a leading zero), every digitally expressed universal difference $[(x+a)^n - x^n]$ is represented by $[(xa)^n - a^n]$. Conversely, if | a,x,y rational, NOT = 0 and n>2**

   a. **when mirrored some putative $(x+a)^n - x^n$ becomes $(ax)^n - a^n$, and not some $(x+y)^n$, then it cannot be an n^{th} power or**

   b. **when mirrored, some putative $(x+a)^n$ becomes $(a+x)^n$ and not some $(xy)^n - a^n$, then it cannot be an n^{th} power difference.**

g) **The constant of the super-*family* $[((x+\alpha)+a)^n]$ is $(\alpha+a)^n$**

h) **The constant of the *family* $[(x+a)^n - x^n]$ is**

$na+n(n+1)a^2/2+\ldots+na^{n-1}+a^n$ ***and can never be zero unless a and (α (& etc)) are zero***

Iff all elements of the domain of both *families* are all rational, then if the only intersection of $[x^n]$ with $[(x+a)^n - x^n]$ when n>2 is when x or a or α or x-a is zero, then **FLT** is proven

In order to more convincingly prove **FLT** (without resorting to tactics unknown to Fermat), consider just the *power differences* ***from*** 1 (1^n) or 0 (0^n) ***to*** any element (x) raised to that same power ***n*** (x^n) which may be represented by x^n-1 or x^n-0, respectively. In each of these cases, **were any power difference $x^n-(0^n)$ with family *constant* $na+n(n+1)a^2/2+\ldots+na^{n-1}+a^n$ to equal the same power of another integer, say y^n**, when a=0 then the constant of $y^n - a^n$ should be **0**. **However,** in that same case the power ***difference constant*** of $y^n-(1^n)$ would be **$na+n(n+1)a^2/2+\ldots+na^{n-1}$ *+ a^n-1* which is incompatible with the family requirement of power differences.**

e.g., if $\mathbf{y^5 = 5a \cdot 10a^2 \cdot 10\ a^3 \cdot 5a^4 \cdot a^5{}_x}$, then

$y = \sqrt[5]{(5a \cdot 10a^2 \cdot 10\ a^3 \cdot 5a^4 \cdot a^5{}_x)}$, ***but*** y^5-a^5 =0 and

$\mathbf{(5a \cdot 10a^2 \cdot 10\ a^3 \cdot 5a^4 \cdot y^5 - a^5)_x = 0}$ and since

$$x^5 = x^5 + 0 \text{ then}$$

$$x^5 = (1 \cdot 5a \cdot 10a^2 \cdot 10\ a^3 \cdot 5a^4 \cdot y^5 - a^5)_x$$

and, therefore either y = a = 0

(or $a^5 - y^5 = (5a \cdot 10a^2 \cdot 10\ a^3 \cdot 5a^4 \cdot 0)_x$ **which is a contradiction to 5.h ABOVE),** *AND SINCE THIS LOGIC FOLLOWS FOR ALL n>2, strongly suggests FLT.*

The take-home point is that the intersection of non-zero based representations of universal powers >2 with that of non-zero based representations of universal differences among these same powers is the null set, or

$$(x+a)^n \cap (x+a)^n - x^n \mid x,a,n \in Q,\ n>2 = \{x=-a, \text{ or } a,x=0\}$$

as the **$(x+a)^n - x^n$ family excludes values with any zero constant** except in the trivial situations mentioned where **{x=-a,or a,x=0}.**

8) A few of the more interesting aspects of ***algebraic families*** follow. The operations of the afore-defined operator ***α***
   a. Leave the highest power coefficient constant with the next highest changing linearly as a first degree function, the next changing as a function in the second degree, and so forth
   b. As may be seen in Section II.3.b above, each subsequent iteration (or row) changes the constant (or χ) to the sum of the coefficients of the immediately preceding (above) row, i.e. if the row directly above were

$$a_1x^2 + b_1x + \chi_1 \text{ , then}$$

$\chi_2 = a_1 + b_1 + \chi_1$, $b_2 = a_1 + 2a$, and $a_2 = a$ and

$\chi_3 = a_2 + b_2 + \chi_2$, $b_2 = a_2 + 2a$, and $a_3 = a$ (&et cetera)

9) Unlike the general category of algebraic integers where the constant term is characterized as an *independent* variable, the constant or constant χ in a **family** is, in fact, a very *dependent* variable. Indeed, the coefficients of each term are fixed by the coefficients of the preceding (domain decreasing by a value of one (1) terms in a fashion that almost rather invents differential calculus:-

Remember any algebraic integer, ζ, in Delinferni format

$$\zeta(x) = C_n \cdot C_{n-1} \cdot C_{n-2} \cdot C_{n-3} \cdot \ldots \cdot C_3 \cdot C_2 \cdot C_1 \cdot C_{0x}$$

C_i of the family δ levels below $[\zeta(x+\delta)]$ can be expressed by the following partial derivatives with respect to δ:-

$$C_0 = \zeta(\delta-1)/0! \; [= C_n+C_{n-1}+C_{n-2}+C_{n-3}+\ldots+C_3+C_2+C_1+C_0 \text{ when } \delta=1]$$

$$C_1 = \zeta'(\delta-1)/1! \; [= (d\zeta(\delta-1)/d\delta)/1]$$

$$C_2 = \zeta''(\delta-1)/2! \; [= (d\zeta'(\delta-1)/d\delta)/2]$$

$$C_3 = \zeta'''(\delta-1)/3! \; [= (d\zeta''(\delta-1)/d\delta)/6]$$

…

$$C_n = [\zeta^{n\prime}(\delta-1)/n! = (d\zeta^{(n-1)\prime}(\delta-1)/d\delta)/n!]$$

(and once again displaying an ontogeny recapitulating phylogeny)

10) Given a successive string of ***at least n+3*** constants (or range values) the entire algebraic integer (polynomial) can be reconstructed. Just the constant term can also differentiate, for example, the algebraic integer for the non-trivial 3rd roots of unity ($x^2 \pm x -1$) from that of ϕ ($x^2 \pm x-1$). If the nucleotides *TdR*, *AdR*, *CdR*, and *GdR* can be assigned values of, say, 0, -1,+1, and *next*, might any given string of DNA not be considered as an algebraic integer to base 3?

11) *The difference among the roots of any given family*

ζ**(x) = $C_n \cdot C_{n-1} \cdot C_{n-2} \cdot C_{n-3} \cdot \ldots \cdot C_3 \cdot C_2 \cdot C_1 \cdot C_{0x}$**

with domain values differing by δ is:- ***$C_n\delta$,***

i.e., if in any given family ζ**(x)** where $(x - x_{1a})(x - x_{1b})\ldots(x - x_{1\omega}) = 0$,

and $(x + \delta - x_{2a})(x + \delta - x_{2b})\ldots(x + \delta - x_{2\omega}) = 0$, then

$(x_{1a} - x_{2a})$ and $(x_{1b} - x_{2b})$...and...$(x_{1\omega} - x_{2\omega}) = C_n\delta$.

IV. The Fibonacci Series

1) Although the **<u>Fundamental Fibonacci Operation</u>** of integral addition has usually been defined *retrospectively* as

$$f_a = f_{a-2} + f_{a-1}$$

it may be described ***<u>prospectively</u>*** by the following stepwise *directional* movement (per step) with two (2) actors, **a** and **b, where**:-

(b-a) ← a → b
a ← b → (a + b)

Fibonacci #	structure	#a	#b	#a + #b
-8	13b – 21a	-21	13	-8
5	13a – 8b	13	-8	5
-3	5b - 8a	-8	5	-3
2	5a – 3b	5	-3	2
-1	2b – 3a	-3	2	-1
1	2a – b	2	-1	1
0	a – b	-1	1	0
1	a	1	0	1
1	b	0	1	1
2	a + b	1	1	2
3	2b + a	1	2	3
5	2a + 3b	2	3	5
8	5b + 3a	3	5	8

(Notice the columnar Fibonacci and the diagonal symmetries.)

2) The extended *family* of Fibonacci numbers can also provide an interesting illustration of *family* inheritance and relationships. This familiar *family* contains ***cousins,*** for example one derived by:

a. Taking the *family* of $\mathbf{x^2 \pm x - 1}$ which includes:

1	**-9**	**19**
1	**-7**	**11**
1	**-5**	**5**
1	**-3**	**1**
1	**-1**	**-1**
1	**1**	**-1**
1	**3**	**1**
1	**5**	**5**
1	**7**	**11**
1	**9**	**19**

b. And, starting with $f_a = 0$, derive a sequence such that

$$f_{a+1}=(f_a\pm(5f_a{}^2+4i^{2a})^{½})/2$$

If

ϕ is a root of $\mathbf{x^2 \pm x - 1}$, then the formula is

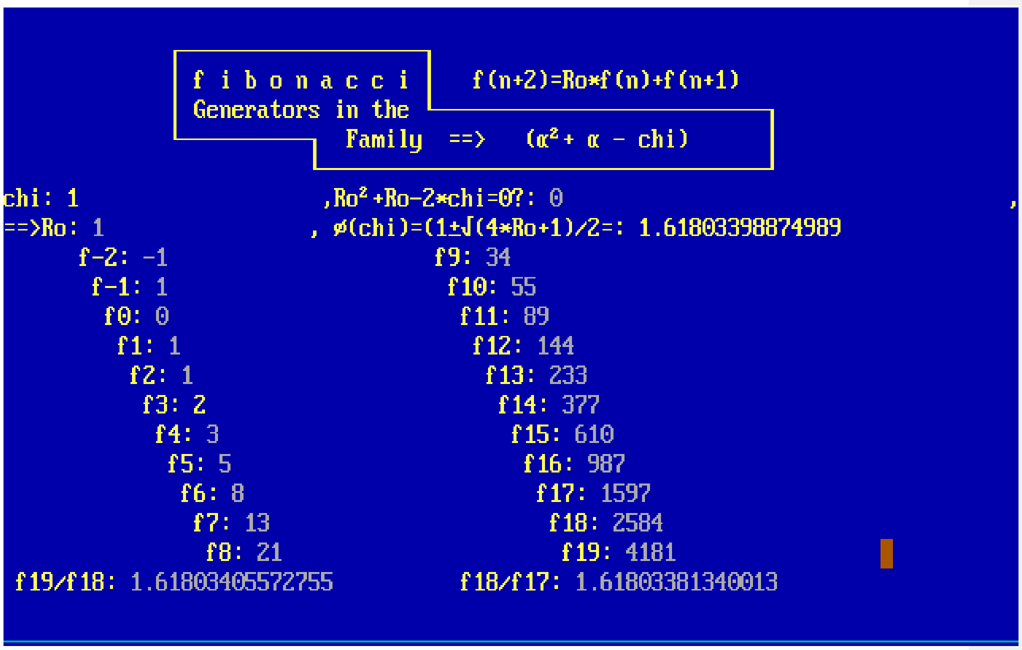
$$f_n =(\phi^n-\phi^{-n}) /(\phi^1+\phi^{-1})$$

it turns out that the ratio of adjacent Fibonacci numbers generated by this formula approaches ϕ as n->∞.

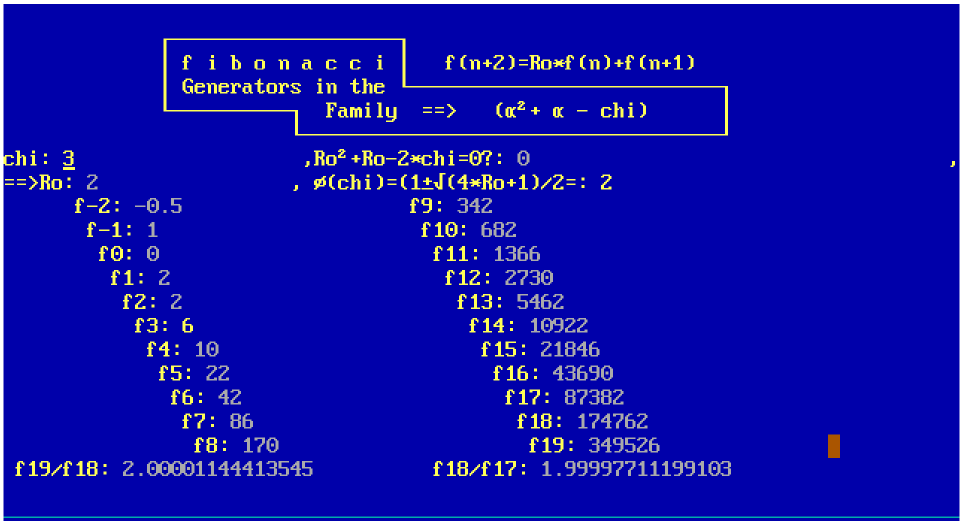
It also turns out that one can change the basic Fibonacci progression by altering the constant from 1 to χ, and that, by so doing, alter the Fibonacci progression by a factor, Ro (or

P) where $\mathbf{f_2} = P\mathbf{f_1} + \mathbf{f_2}$ and $P^2 + P - 2\chi = 0$

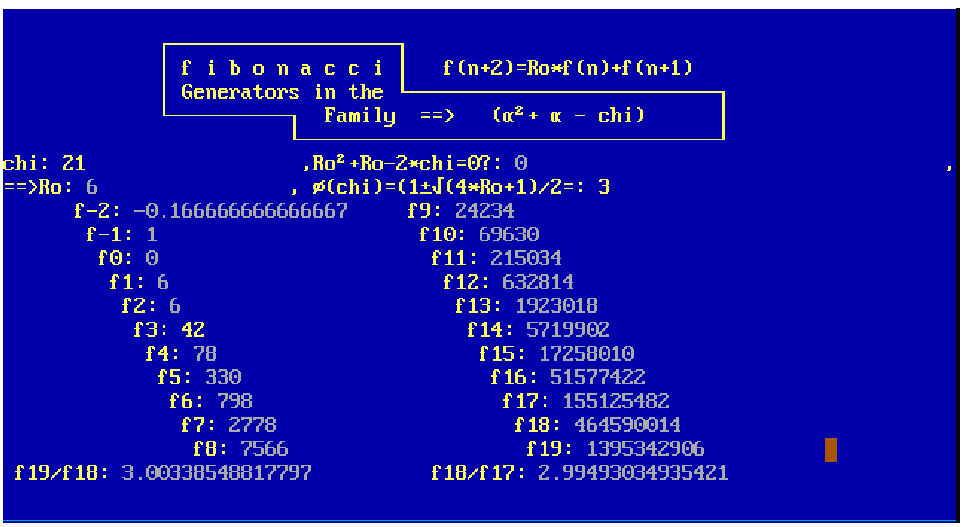
Standard Fibonacci with χ **=1 and** P =1



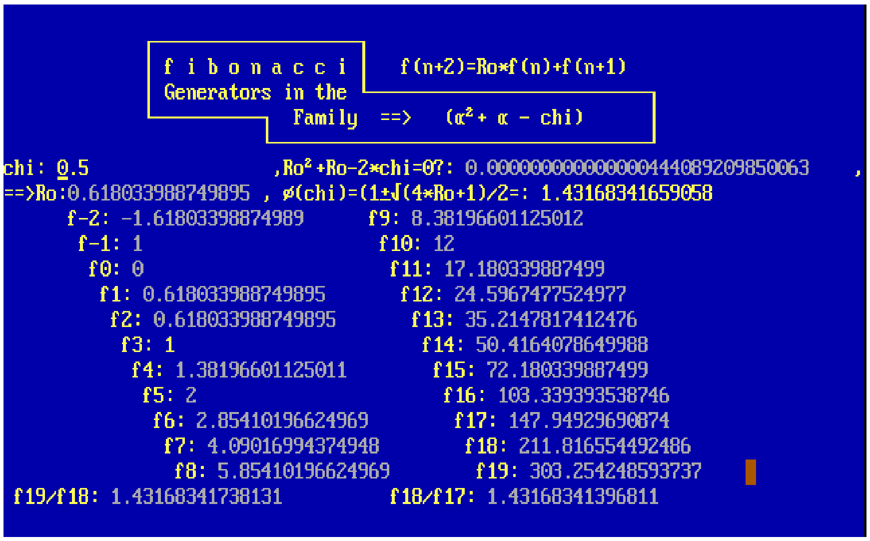
Fibonacci cousin with χ **=3 and** P =2



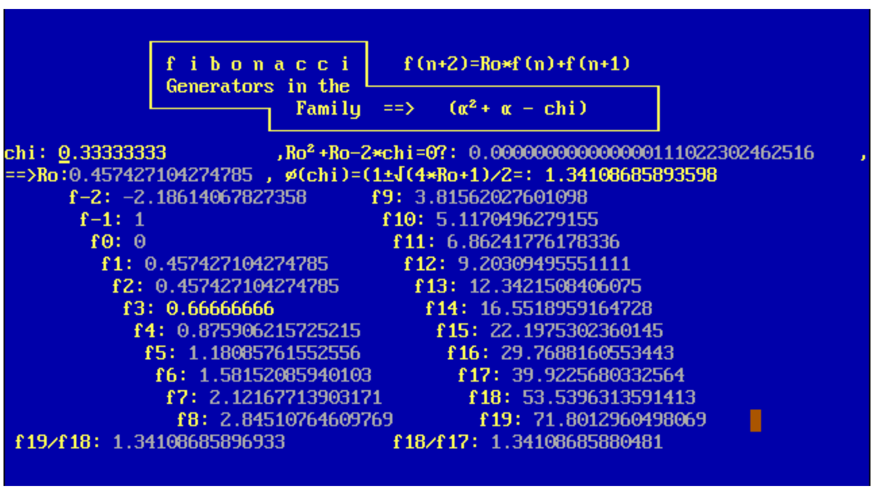
Fibonacci cousin with χ **=21 and** P = 6



Fibonacci cousin with χ **=0.5 and** P = ϕ



Fibonacci cousin with χ **=1/3 and** P = 1+ϕ/2



It follows form this indexing that:-

a. The ***odd***-*indexed* Fibonacci numbers are always **positive** (in this sequence)

b. The ***even***-indexed Fibonacci numbers are always **<u>negative</u>** when the **integral indices are less than zero** and always **<u>positive</u>** when the **integral indices are greater than zero** (in this sequence)

c. The Fibonacci numbers corresponding to indices which are 0 mod y are all themselves congruent to 0 mod f_y

$$\mathbf{o_{f\ mod(y)} = 0\ mod(f_y)}$$

(It follows that since f_5 = 5, all Fibonacci numbers whose indices are 0 mod 5 are themselves all congruent to 0 mod 5.)

d. All Fibonacci numbers mod any integer are cyclic. The cycle for mod 5 is 20 (1,1,2,3,0,3,3,1,4,0,4,4,3,2,0,2,2,4,1,0). **The Fibonacci numbers f_y whose indices are 0 mod y are not all congruent to 0 mod y**, **but they are all also congruent to 0 modulo their cycle length which in turn has f_y as a factor.**

II. The **<u>Fundamental Indexed Fibonacci Operation</u>** is integral addition and is defined as follows:-

$$f_{a+b} = f_a f_b + f_{a-1} f_b + f_a f_{b-1}$$

III. It should be fairly obvious that the integral Fibonacci indices form a group under addition as the group operation with "0" as the identity

a. It may be less obvious, but the ***<u>even</u>-indexed*** Fibonacci ***numbers*** themselves also form a group under addition with each number having its negative as inverse.

b. It also should be obvious that the ***<u>odd</u>-indexed*** Fibonacci ***numbers*** themselves do not form a group under addition

IV. The <u>sum of the squares of two contiguous Fibonacci</u> is another Fibonacci number which is indexed ***odd***:-

$$f_a^2 + f_{a-1}^2 = f_{a+a-1} = f_{2a-1}$$

$$f_a^2 + f_{a+1}^2 = f_{a+a+1} = f_{2a+1}$$

V. The difference between <u>the squares of two semi-adjacent Fibonacci</u> is another Fibonacci number which is indexed ***even***:-

$$f_{a+1}^2 - f_{a-1}^2 = f_{2a}$$

VI. It is also true for any **2 contiguous Fibonacci numbers $f_{1'}$ and $f_{2'}$** that:

$$(f_{2'}+1)(f_{2'}-1) = f_{1'}(f_{1'} + f_{2'})$$

$$f_{2'}^2-1 = (f_{1'}^2 + f_{1'}f_{2'})$$

$$f_{2'}^2 - f_{1'}f_{2'} - (f_1'^2+1) = 0$$

ergo, using a modified quadratic formula

$$f_2 = \frac{f1 \pm \sqrt{f1^2+4f1^2 \pm 4}}{2}$$

$$= \frac{f1 \pm \sqrt{5f1^2 \pm 4}}{2}$$

or

$$f_b = (f_a \pm (5f_a^2+4i^{2a})^{½})/2$$

Following is a table of values when the sign for 4 is + or - :-

a	f_1	sqrt($5f_1^2+4$)	sqrt($5f_1^2-4$)
-6	-8	±18	±sqrt(18^2-8)
-5	5	±sqrt(11^2+8)	±11
-4	-3	±7	±sqrt(7^2-8)
-3	2	±sqrt(4^2+8)	±4
-2	-1	±3	±sqrt(3^2-8)
-1	1	±sqrt(1^2+8)	±1
0	0	±2	±sqrt(2^2-8)
1	1	±sqrt(1^2+8)	±1
2	1	±3	±sqrt(3^2-8)
3	2	±sqrt(4^2+8)	±4
4	3	±7	±sqrt(7^2-8)
5	5	±sqrt(11^2+8)	±11
6	8	±18	±sqrt(18^2-8)
7	13	±sqrt(29^2+8)	±29
8	21	±47	±sqrt(47^2-8)
9	34	±sqrt(76^2+8)	±76
10	55	±123	±sqrt(123^2-8)
11	89	±sqrt(199^2+8)	±199
12	144	±322	±sqrt(322^2-8)
13	233	±sqrt(521^2+8)	±521
14	377	±843	±sqrt(843^2-8)
15	610	±sqrt(1364^2+8)	±1364
16	987	±2207	±sqrt(2207^2-8)

VII. The sum of the ***cubes*** of two contiguous Fibonacci numbers minus that of the immediately preceding Fibonacci number is also another Fibonacci number which is always indexed ***zero mod 3***:-

$$f_a^3 + f_{a+1}^3 - f_{a-1}^3 = f_{a+a+a} = f_{3a}$$

n.b., it is also true that:-

$$f_{3a} = f_a((f_{2a+1} + f_{2a-1}) + i^{2a}) = f_{a+a+a}$$

VIII. The indexed integers for the Fibonacci numbers do not form a group under multiplication as fractional indices are not defined (as yet.) Yet the indices do form an integral ring with identity under addition and multiplication. The operational law for indexed Fibonacci multiplication is defined in terms of odd and even indexed numbers, i.e.,

a. If ***y is odd***, then:-

$$f_{ya} = f(_{ya-1)/2})^2 + f(_{ya+1)/2})^2$$

b. If ***y is even***, then:-

$$f_{ya} = f(_{ya-2)/2})^2 + f(_{ya+2)/2})^2$$

c. Additional multiplication formulas for 2a, 3a have already been given, but here is one for 5a:-

$$f_{5a} = ((f_{3a+1} + f_{3a-1})(2a-1)+1)f_a$$

IX. The ratio of any two successive Fibonacci numbers, f_x and f_{x+1} approaches the ***Golden Ratio, Φ,*** as x approaches infinity.

$$(1-\boldsymbol{\Phi})/\boldsymbol{\Phi} = \boldsymbol{\Phi}/1$$

$$\boldsymbol{\Phi}^2 = 1 - \boldsymbol{\Phi}$$

$$\boldsymbol{\Phi}^2 + \boldsymbol{\Phi} - 1 = 0$$

$$\boldsymbol{\Phi} = (-1 \pm \mathrm{sqrt}(5))/2$$

i.e., ***Φ*** lies at the midpoint between 1 and the square roots of 5

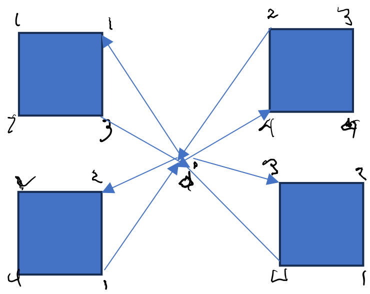
X. Note also that as $f_a = f_{a+b} - f_{a-b}$ for b = odd, and

$f_a = f_{a+b} + f_{a-b}$ for b = even, then

$$f_a = f_{a+b} + f_{a-b}i^{2b}, \text{ and}$$

$$f_a = f_a\,(f_{b+2} + f_{b-2}i^{2b})$$

XI. Since the Fibonacci Series modula 5 is a 20 cycle*, its structure could be represented geometrically one way as follows:



(which could, conceivably, represent four limbs attached to a corpus)

* 1,1,2,3,0,3,3,1,4,0,4,4,3,2,0,2,2,4,1,0

IV. The Tsadik Function (צ)

In yet another digression, look at what happens when

if $x^n + y^n = z^n$, **b = x - y** and **a = z - x**, we divide

$((x + a)^n – x^n)$ or y^n by **(x+b)** or y - which should be 1 -

and examine the [Fermat] residue $((x + a)^n – x^n) \bmod (b + a)$ - which should be 0

and which I shall call "Tsadik", i.e.,

$$צ = ((x + a)^n – x^n) \bmod (x + b)$$

$$צ = n! \Sigma_{\text{from } x=0 \text{ to } n-1} ((a^{n-x}(-b)^x)/(x!(n-x)!))$$

$$צ_{n=1} = a$$

$$צ_{n=2} = a^2 – 2ab$$

$$צ_{n=3} = a^3 – 3a^2b + 3ab^2$$

צ $_{n=5} = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4$

צ $_{n=7} = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6$

I have essentially 3 conjectures here:-

1) If, when each complementary pair of the imaginary roots is considered once, the number of unique non-trivial roots of צ $_n$ is ***(n-1)/2***, then n is prime

2) The real coefficient of these non-trivial roots when solving for ***b*** is always equal to ***a/2*** (Riemann relationship here?)

3) The only real solutions exist when a=b=0 or x=0 and a=b or x=-b

Some examples solving for b when צ $_n = 0$:

n	a	b
5	-6	-3 ± 0.9748i, -3 ± 4.1291i
	+6	+3 ± 0.9748i, +3 ± 4.1291i
	12	+6 ± 1.9495i, +6 ± 8.2583i
	18	+9 ± 2.9243i, +9 ± 12.387i
	108	54 ± 17.546i, 54 ± 74.325i
7	-6	-3 ± 6.2296i, -3 ± 2.3924i, -3-0.6847i
	6	3 ± 6.2296i, 3 ± 2.3924i, 3-0.6847i
	12	6 ± 12.4591i, 6 ± 4.7848i, 6 -1.3695i

In S U M M A R Y

2. *2-diffs* and *2-powers* as defined are indistinct sets.
3. The only *2-diffs* that are *2-powers* also obey:-

   a. $(2a \cdot a^2{}_x)^2 + (2a \cdot 2a^2 \cdot (a^3 - a)/2_x)^2 = (2a \cdot 2a^2 \cdot (a^3 + a)/2_x)^2$

4. $k((21_x)^2 + (220_x)^2 = (221_x)^2)$

5. **All *n-powers* must therefore obey:-**

   a. $((2a \cdot a^2{}_x)^{2/n})^n + ((2a \cdot 2a^2 \cdot (a^3 - a)/2_x)^{2/n})^n = ((2a \cdot 2a^2 \cdot (a^3 + a)/2_x)^{2/n})^n$

   Or $k(((21_x)^{2/n})^n + ((220_x)^{2/n})^n = ((221_x)^{2/n})^n)$

   b. These both lead to at least 1 irrational base or coefficient when n>2 since

   $(a^6 - 2a^4 + a^2)/4_x + (4a^4)/4_x = (a^6 + 2a^4 + a^2)/4_x$

   which does not apply when n>2 as:-

   $(a^{n1})^{2}+((a^{n2}-a^{n3})/2)^2 = ((a^{n2}+a^{n3})/2)^2$

   only applies as long as:-

   i. n1=(n2+n3)/2, and
   ii. the additional mid-powered (e.g., a^4 when n=2) terms can cancel out, which they do not when n1>2
   iii. Solve (x+1)^n=x^n + (x-1)^n

   n:2 x = 4

   n:3. x = 2+1/3 (243-27 sqrt(17))^(1/3) + (9+sqrt(17))^(1/3) near x~=6.0546

   n:4 x = 1/3 (8+(620-12 sqrt(849))^(1/3)+2^(2/3) (155+3 sqrt(849))^(1/3)) near x~= 8.1213

   n:5 x = root of x^5-10 x^4-20 x^2-2 near x ~=10.1927

   n:6. x = root of x^5-12 x^4-40 x^2-12 near x ~= 12.2664

   n:7 x = root of x^7-14 x^6-70 x^4-42 x^2-2 near x ~= 14.3413

n:8 x = root of x^7-16 x^6-112 x^4-112 x^2-16
near x ~= 16.4171
n:9 x = root of x^9-18 x^8-168 x^6-252 x^4-72 x^2-2
near x ~= 18.4934
n:10 x = root of x^9-20 x^8-240 x^6-504 x^4-240 x^2-20 near x ~= 20.57
n:20 x = root of x^19-40 x^18-2280 x^16-31008 x^14-155040 x^12-335920 x^10-335920 x^8-155040 x^6-31008 x^4-2280 x^2-40 near x ~= 41.3445
n:25 near ~=51.7336
n:50 x = root of x^49-100 x^48-39200 x^46-4237520 x^44-199768800 x^42-5010867400 x^40-74707477600 x^38-709721037200 x^36-4501659150240 x^34-19694758782300 x^32-60811886766400 x^30-134654892125600 x^28-216086506731200 x^26-252821212875504 x^24-216086506731200 x^22-134654892125600 x^20-60811886766400 x^18-19694758782300 x^16-4501659150240 x^14-709721037200 x^12-74707477600 x^10-5010867400 x^8-199768800 x^6-4237520 x^4-39200 x^2-100
(near x ~= 103.683)

Show x →2n+ ln(n/2) with $\delta = \ln(n)$ irrational for all $(x-1)^n + x^n = (x+1)^n$

6. Since FLT has been proven for all even powers (2n | n integer >1) then:-

$$((2a \cdot a^2{}_x)^{1/n})^{2n} + ((2a \cdot 2a^2 \cdot (a^3 - a)/2_x)^{1/n})^{2n} = ((2a \cdot 2a^2 \cdot (a^3 + a)/2_x)^{1/n})^{2n}$$

Or $k(((21_x)^{1/n})^{2n} + ((220_x)^{1/n})^{2n} = ((221_x)^{1/n})^{2n})$ **proves FLT for all**

n=2n+1 (for all odd n integers odd)

7. *n-diffs* and *n-* of n>2 with rational elements as defined are, therefore, distinct sets with only intersection {0,1,-1}.
8. Under multiplication, all *n-powers* take their *n-powers* to *n-powers*
9. All *n-powers* > 2 take their *n-diffs* to *n-diffs* and their *n-powers* to *n-powers* under multiplication.
10. Under multiplication, all *2-powers* take their
    a. *2-powers* to *2-powers*
    b. *2-diffs* that are also *2-powers* to *2-powers* and
    c. *2-diffs* that are not also *2-powers* to *2-diffs*
11. Therefore if under multiplication an *n-power* takes an *n-diff* to an *n-power* then:-

n must =2 **[Q.E.D #1]**

-A D D E N D U M 1 -

Families of $x^2 + xy + y^2$, when x=1 and:-

y=1 y=2 y=3 y=4 y=5 y=6

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
-8	1	-17	73	1	-16	67	1	-15	63	1	-14	61	1	-13	61	1	-12	63
-7	1	-15	57	1	-14	52	1	-13	49	1	-12	48	1	-11	49	1	-10	52
-6	1	-13	43	1	-12	39	1	-11	37	1	-10	37	1	-9	39	1	-8	43
-5	1	-11	31	1	-10	28	1	-9	27	1	-8	28	1	-7	31	1	-6	36
-4	1	-9	21	1	-8	19	1	-7	19	1	-6	21	1	-5	25	1	-4	31
-3	1	-7	13	1	-6	12	1	-5	13	1	-4	16	1	-3	21	1	-2	28
-2	1	-5	7	1	-4	7	1	-3	9	1	-2	13	1	-1	19	1	0	27
-1	1	-3	3	1	-2	4	1	-1	7	1	0	12	1	1	19	1	2	28
0	1	-1	1	1	0	3	1	1	7	1	2	13	1	3	21	1	4	31
1	1	1	1	1	2	4	1	3	9	1	4	16	1	5	25	1	6	36
2	1	3	3	1	4	7	1	5	13	1	6	21	1	7	31	1	8	43
3	1	5	7	1	6	12	1	7	19	1	8	28	1	9	39	1	10	52
4	1	7	13	1	8	19	1	9	27	1	10	37	1	11	49	1	12	63
5	1	9	21	1	10	28	1	11	37	1	12	48	1	13	61	1	14	76
6	1	11	31	1	12	39	1	13	49	1	14	61	1	15	75	1	16	91
7	1	13	43	1	14	52	1	15	63	1	16	76	1	17	91	1	18	108
8	1	15	57	1	16	67	1	17	79	1	18	93	1	19	109	1	20	127
9	1	17	73	1	18	84	1	19	97	1	20	112	1	21	129	1	22	148
10	1	19	91	1	20	103	1	21	117	1	22	133	1	23	151	1	24	171
11	1	21	111	1	22	124	1	23	139	1	24	156	1	25	175	1	26	196
12	1	23	133	1	24	147	1	25	163	1	26	181	1	27	201	1	28	223
13	1	25	157	1	26	172	1	27	189	1	28	208	1	29	229	1	30	252
14	1	27	183	1	28	199	1	29	217	1	30	237	1	31	259	1	32	283
15	1	29	211	1	30	228	1	31	247	1	32	268	1	33	291	1	34	316
16	1	31	241	1	32	259	1	33	279	1	34	301	1	35	325	1	36	351
17	1	33	273	1	34	292	1	35	313	1	36	336	1	37	361	1	38	388
18	1	35	307	1	36	327	1	37	349	1	38	373	1	39	399	1	40	427
19	1	37	343	1	38	364	1	39	387	1	40	412	1	41	439	1	42	468
20	1	39	381	1	40	403	1	41	427	1	42	453	1	43	481	1	44	511
21	1	41	421	1	42	444	1	43	469	1	44	496	1	45	525	1	46	556
22	1	43	463	1	44	487	1	45	513	1	46	541	1	47	571	1	48	603
23	1	45	507	1	46	532	1	47	559	1	48	588	1	49	619	1	50	652
24	1	47	553	1	48	579	1	49	607	1	50	637	1	51	669	1	52	703
25	1	49	601	1	50	628	1	51	657	1	52	688	1	53	721	1	54	756
26	1	51	651	1	52	679	1	53	709	1	54	741	1	55	775	1	56	811

-A D D E N D U M 2 –

x	x^2 *? + 0	=	(x^2 + x + 1)*? + 1	=	(x^2 + 2x + 4)*? + 8	=	(x^2 + 3x + 9)*? + 27	=	(x^2 + 4x + 16)*? +64	=	(x^2 + 5x + 25)*? + 125	=
-1	-1	-1	-2	-1	-3	-1	-4	-1	-5	-1	-6	-1
0	0	0	-1	0	-2	0	-3	0	-4	0	-5	0
1	1	1	0	1	-1	1	-2	1	-3	1	-4	1
2	2	8	1	8	0	8	-1	8	-2	8	-3	8
3	3	27	2	27	1	27	0	27	-1	27	-2	27
4	4	64	3	64	2	64	1	64	0	64	-1	64
5	5	125	4	125	3	125	2	125	1	125	0	125
6	6	216	5	216	4	216	3	216	2	216	1	216
7	7	343	6	343	5	343	4	343	3	343	2	343
8	8	512	7	512	6	512	5	512	4	512	3	512
9	9	729	8	729	7	729	6	729	5	729	4	729
10	10	1000	9	1000	8	1000	7	1000	6	1000	5	1000
11	11	1331	10	1331	9	1331	8	1331	7	1331	6	1331
12	12	1728	11	1728	10	1728	9	1728	8	1728	7	1728
13	13	2197	12	2197	11	2197	10	2197	9	2197	8	2197
14	14	2744	13	2744	12	2744	11	2744	10	2744	9	2744
15	15	3375	14	3375	13	3375	12	3375	11	3375	10	3375
16	16	4096	15	4096	14	4096	13	4096	12	4096	11	4096

$$(x^2+ax+a^2)(x-1) + a^3 = x^3$$

-A D D E N D U M 3 –

(for any base **x** where $\mathbf{X^2+Y^2=Z^2}$ and $\mathbf{a=Z_0-Y_0}$):-

$$\mathbf{(2a \cdot a^2{}_{x0})^2 + (2a \cdot 2a^2 \cdot (a^3 - a)/2_{x0})^2 = (2a \cdot 2a^2 \cdot (a^3 + a)/2_{x0})^2}$$

when $Z_1 - Y_1 = \mathbf{1}$**, then**

$$\mathbf{k((21_{x1})^2 + (220_{x1})^2 = (221_{x1})^2)}$$

substituting $\mathbf{k=a^{-1}}$ and $\mathbf{x_1=x_0+(a-1)/2}$:-

a	x_0	X_0=2ax_0+a^2	Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2	Pyth <=> Z_1-Y_1=1	Divisor (=a)	x_1=>x_0+(a-1)/2
1	1	2+1=3	2+2+0=4	**3 4 5**	1	1
2	1	4+4=8	4+8+3=15	**4 7.5 8.5**	2	1.5
3	1	6+9=15	6+18+12=36	**5 12 13**	3	2
4	1	8+16=24	8+32+30=70	**6 17.5 18.5**	4	2.5
5	1	10+25=35	10+50+60=120	**7 24 25**	5	3
6	1	12+36=48	12+72+105=189	**8 31.5 32.5**	6	3.5
7	1	14+49=63	14+98+168=280	**9 40 41**	7	4
8	1	16+64-80	16+128+252=396	**10 49.5 50.5**	8	4.5
9	1	18+81=99	18+162+360=540	**11 60 61**	9	5
10	1	20+100=120	20+200+495=715	**12 71.5 72.5**	10	5.5
11	1	22+121=143	22+242+660=924	**13 84 85**	11	6
12	1	24+144=168	24+288+858=1170	**14 97.5 98.5**	12	6.5
13	1	26+169=195	26+338+1092=1456	**15 112 113**	13	7
14	1	28+196=224	28+392+1365=1785	**16 127.5 128.5**	14	7.5
15	1	30+225=255	30+450+1680=2160	**17 144 145**	15	8
16	1	32+256=288	32+512+2040=2584	**18 161.5 162.5**	16	8.5
17	1	34+289=323	34+578+2448=3060	**19 180 181**	17	9
18	1	36+324=360	36+648+2907=3591	**20 199.5 200.5**	18	9.5
19	1	38+361=399	38+722+3420=4180	**21 220 221**	19	10
20	1	40+400=440	40+800+3990=4830	**22 241.5 242.5**	20	10.5
21	1	42+441=483	42+882+4620=5544	**23 264 265**	21	11
22	1	44+484=528	44+968+5313=6325	**24 287.5 288.5**	22	11.5

a	x_0	X_0=2ax_0+a^2	Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2	Pyth <=> Z_1-Y_1=1	Divisor (=a)	x_1=>x_0+(a-1)/2
1	1.5	3+1=4	4.5+3+0=7.5	**4 7.5 8.5**	1	1.5
2	1.5	6+4=10	9+12+3=24	**5 12 13**	2	2
3	1.5	9+9=18	13.5+27+12=52.5	**6 17.5 18.5**	3	2.5
4	1.5	12+16=28	18+48+30=96	**7 24 25**	4	3
5	1.5	15+25=40	22.5+75+60=157.5	**8 31.5 32.5**	5	3.5
6	1.5	18+36=54	27+108+105=240	**9 40 41**	6	4
7	1.5	21+49=70	31.5+147+168=346.5	**10 49.5 50.5**	7	4.5
8	1.5	24+64-88	36+192+252=480	**11 60 61**	8	5
9	1.5	27+81=108	40.5+243+360=643.5	**12 71.5 72.5**	9	5.5
10	1.5	30+100=130	45+300+495=840	**13 84 85**	10	6
11	1.5	33+121=154	49.5+363+660=1072.5	**14 97.5 98.5**	11	6.5
12	1.5	36+144=180	54+432+858=1170	**15 112 113**	12	7
13	1.5	39+169=208	58.5+507+1092=1738.5	**16 127.5 128.5**	13	7.5
14	1.5	42+196=238	63+588+1365=2016	**17 144 145**	14	8
15	1.5	45+225=270	67.5+675+1680=2422.5	**18 161.5 162.5**	15	8.5
16	1.5	48+256=304	72+768+2040=2860	**19 180 181**	16	9
17	1.5	51+289=340	76.5+867+2448=3391.5	**20 199.5 200.5**	17	9.5
18	1.5	54+324=378	81+972+2907=3960	**21 220 221**	18	10
19	1.5	57+361=418	85.5+1083+3420=4588.5	**22 241.5 242.5**	19	10.5
20	1.5	60+400=460	90+1200+3990=5280	**23 264 265**	20	11
21	1.5	63+441=504	94.5+1323+4620=6037.5	**24 287.5 288.5**	21	11.5
22	1.5	66+484=550	99+1452+5313=6864	**25 312 313**	22	12

a	x_0	X_0=2ax_0+a^2	Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2	Pyth <=> Z_1-Y_1=1	Divisor (=a)	x_1=>x_0+(a-1)/2
1	2	4+1=5	8+4+0=12	**5 12 13**	1	2
2	2	8+4=12	16+16+3=35	**6 17.5 18.5**	2	2.5
3	2	12+9=21	24+36+12=72	**7 24 25**	3	3
4	2	16+16=32	32+64+30=126	**8 31.5 32.5**	4	3.5
5	2	20+25=45	40+100+60=200	**9 40 41**	5	4
6	2	24+36=60	48+144+105=297	**10 49.5 50.5**	6	4.5
7	2	28+49=77	56+196+168=420	**11 60 61**	7	5
8	2	32+64-96	64+256+252=572	**12 71.5 72.5**	8	5.5
9	2	36+81=117	72+324+360=688	**13 84 85**	9	6
10	2	40+100=140	80+400+495=975	**14 97.5 98.5**	10	6.5
11	2	44+121=165	88+484+660=1232	**15 112 113**	11	7
12	2	48+144=192	96+576+858=1530	**16 127.5 128.5**	12	7.5
13	2	52+169=221	104+676+1092=1872	**17 144 145**	13	8
14	2	56+196=252	112+784+1365=2261	**18 161.5 162.5**	14	8.5
15	2	60+225=285	120+900+1680=2700	**19 180 181**	15	9
16	2	64+256=320	128+1024+2040=3192	**20 199.5 200.5**	16	9.5
17	2	68+289=357	136+1156+2448=3740	**21 220 221**	17	10
18	2	72+324=396	144+1296+2907=4347	**22 241.5 242.5**	18	10.5
19	2	76+361=437	152+1444+3420=5016	**23 264 265**	19	11
20	2	80+400=480	160+1600+3990=5750	**24 287.5 288.5**	20	11.5
21	2	84+441=525	168+1764+4620=6552	**25 312 313**	21	12
22	2	88+484=572	176+1936+5313=7425	**26 337.5 338.5**	22	12.5

a	x_0	$X_0=2ax_0+a^2$	$Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2$	Pyth <=> $Z_1-Y_1=1$	Divisor (=a)	$x_1=>x_0+(a-1)/2$
1	3	6+1=7	18+6+0=24	**7 24 25**	1	3
2	3	12+4=16	36+24+3=63	**8 31.5 32.5**	2	3.5
3	3	18+9=27	54+54+12=120	**9 40 41**	3	4
4	3	24+16=40	72+96+30=198	**10 49.5 50.5**	4	4.5
5	3	30+25=55	90+150+60=300	**11 60 61**	5	5
6	3	36+36=72	108+216+105=429	**12 71.5 72.5**	6	5.5
7	3	42+49=63	126+294+168=588	**13 84 85**	7	6
8	3	48+64-80	144+384+252=780	**14 97.5 98.5**	8	6.5
9	3	54+81=99	162+486+360=1008	**15 112 113**	9	7
10	3	60+100=160	180+600+495=1275	**16 127.5 128.5**	10	7.5
11	3	66+121=187	198+726+660=1584	**17 144 145**	11	8
12	3	72+144=216	216+864+858=1938	**18 161.5 162.5**	12	8.5
13	3	78+169=247	234+1014+1092=2340	**19 180 181**	13	9
14	3	84+196=280	252+1176+1365=2793	**20 199.5 200.5**	14	9.5
15	3	90+225=315	270+1350+1680=3300	**21 220 221**	15	10
16	3	96+256=352	288+1536+2040=3864	**22 241.5 242.5**	16	10.5
17	3	102+289=391	306+1734+2448=4488	**23 264 265**	17	11
18	3	108+324=432	324+1944+2907=5175	**24 287.5 288.5**	18	11.5
19	3	114+361=475	342+2166+3420=5928	**25 312 313**	19	12
20	3	120+400=520	360+2400+3990=6750	**26 337.5 338.5**	20	12.5
21	3	126+441=567	378+2646+4620=7644	**27 364 365**	21	13
22	3	132+484=616	396+2904+5313=8613	**28 391.5 392.5**	22	13.5