Fields and Fermat

$\mathbb{Z}$ $\mathbb{Q}$ `$\mathbb{R}$ $\mathbb{C}$

-The Delinferni Fermat Theorem-

The simplest explanation why $\mathbf{X^n + Y^n = Z^n}$ for $\mathbf{n > 2}$ cannot include all non-zero **X, Y, Z** ϵ$\mathbb{Z}$ **or** ϵ$\mathbb{Q}$ `is that when **n>2,** ***X,Y, and Z cannot all be represented by algebraic integers in base x*** as they can easily be when $\mathbf{n \leq 2}$.

1)What is it specifically about n = 2 that allows for $\mathbf{x^n + y^n = z^n}$ to prescribe sets which include all non-zero **x, y, z** ϵ $\mathbb{Z}$?

a. For any base **x** where $\mathbf{X^2+Y^2=Z^2}$ and $\mathbf{a=Z_0-Y_0}$

$$\mathbf{(2a \cdot a^2{}_{x0})^2 + (2a \cdot 2a^2 \cdot (a^3 - a)/2_{x0})^2 = (2a \cdot 2a^2 \cdot (a^3 + a)/2_{x0})^2}$$

And when $\mathbf{Z_1 - Y_1 = 1}$, substituting

$\mathbf{k=a^{-1}}$ and $\mathbf{x_1=x_0+(a-1)/2}$, then :-

b. $\mathbf{k((21_{x1})^2 + (220_{x1})^2 = (221_{x1})^2)}$

2) **All rational instances may therefore be inferred from:-**

a	x_0	X_0=2ax_0+a^2	Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2	Pyth <=> Z_1-Y_1=1	Divisor (=a)	x_1=>x_0+(a-1)/2
1	3	6+1=7	18+6+0=24	**7 24 25**	1	3
2	3	12+4=16	36+24+3=63	**8 31.5 32.5**	2	3.5
3	3	18+9=27	54+54+12=120	**9 40 41**	3	4
4	3	24+16=40	72+96+30=198	**10 49.5 50.5**	4	4.5
5	3	30+25=55	90+150+60=300	**11 60 61**	5	5
6	3	36+36=72	108+216+105=429	**12 71.5 72.5**	6	5.5
7	3	42+49=63	126+294+168=588	**13 84 85**	7	6
8	3	48+64-80	144+384+252=780	**14 97.5 98.5**	8	6.5
9	3	54+81=99	162+486+360=1008	**15 112 113**	9	7
10	3	60+100=160	180+600+495=1275	**16 127.5 128.5**	10	7.5
11	3	66+121=187	198+726+660=1584	**17 144 145**	11	8
12	3	72+144=216	216+864+858=1938	**18 161.5 162.5**	12	8.5
13	3	78+169=247	234+1014+1092=2340	**19 180 181**	13	9
14	3	84+196=280	252+1176+1365=2793	**20 199.5 200.5**	14	9.5
15	3	90+225=315	270+1350+1680=3300	**21 220 221**	15	10
16	3	96+256=352	288+1536+2040=3864	**22 241.5 242.5**	16	10.5
17	3	102+289=391	306+1734+2448=4488	**23 264 265**	17	11
18	3	108+324=432	324+1944+2907=5175	**24 287.5 288.5**	18	11.5
19	3	114+361=475	342+2166+3420=5928	**25 312 313**	19	12
20	3	120+400=520	360+2400+3990=6750	**26 337.5 338.5**	20	12.5
21	3	126+441=567	378+2646+4620=7644	**27 364 365**	21	13
22	3	132+484=616	396+2904+5313=8613	**28 391.5 392.5**	22	13.5

a	x_0	X_0=2ax_0+a^2	Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2	Pyth <=> Z_1-Y_1=1	Divisor (=a)	x_1=>x_0+(a-1)/2
1	1.5	3+1=4	4.5+3+0=7.5	**4 7.5 8.5**	1	1.5
2	1.5	6+4=10	9+12+3=24	**5 12 13**	2	2
3	1.5	9+9=18	13.5+27+12=52.5	**6 17.5 18.5**	3	2.5
4	1.5	12+16=28	18+48+30=96	**7 24 25**	4	3
5	1.5	15+25=40	22.5+75+60=157.5	**8 31.5 32.5**	5	3.5
6	1.5	18+36=54	27+108+105=240	**9 40 41**	6	4
7	1.5	21+49=70	31.5+147+168=346.5	**10 49.5 50.5**	7	4.5
8	1.5	24+64-88	36+192+252=480	**11 60 61**	8	5
9	1.5	27+81=108	40.5+243+360=643.5	**12 71.5 72.5**	9	5.5
10	1.5	30+100=130	45+300+495=840	**13 84 85**	10	6
11	1.5	33+121=154	49.5+363+660=1072.5	**14 97.5 98.5**	11	6.5
12	1.5	36+144=180	54+432+858=1170	**15 112 113**	12	7
13	1.5	39+169=208	58.5+507+1092=1738.5	**16 127.5 128.5**	13	7.5
14	1.5	42+196=238	63+588+1365=2016	**17 144 145**	14	8
15	1.5	45+225=270	67.5+675+1680=2422.5	**18 161.5 162.5**	15	8.5
16	1.5	48+256=304	72+768+2040=2860	**19 180 181**	16	9
17	1.5	51+289=340	76.5+867+2448=3391.5	**20 199.5 200.5**	17	9.5
18	1.5	54+324=378	81+972+2907=3960	**21 220 221**	18	10
19	1.5	57+361=418	85.5+1083+3420=4588.5	**22 241.5 242.5**	19	10.5
20	1.5	60+400=460	90+1200+3990=5280	**23 264 265**	20	11
21	1.5	63+441=504	94.5+1323+4620=6037.5	**24 287.5 288.5**	21	11.5
22	1.5	66+484=550	99+1452+5313=6864	**25 312 313**	22	12

a	x_0	X_0=2ax_0+a^2	Y_0=2a*x_0^2+2a^2*x_0+(a^3-a)/2	Pyth <=> Z_1-Y_1=1	Divisor (=a)	x_1=>x_0+(a-1)/2
1	2	4+1=5	8+4+0=12	**5 12 13**	1	2
2	2	8+4=12	16+16+3=35	**6 17.5 18.5**	2	2.5
3	2	12+9=21	24+36+12=72	**7 24 25**	3	3
4	2	16+16=32	32+64+30=126	**8 31.5 32.5**	4	3.5
5	2	20+25=45	40+100+60=200	**9 40 41**	5	4
6	2	24+36=60	48+144+105=297	**10 49.5 50.5**	6	4.5
7	2	28+49=77	56+196+168=420	**11 60 61**	7	5
8	2	32+64-96	64+256+252=572	**12 71.5 72.5**	8	5.5
9	2	36+81=117	72+324+360=688	**13 84 85**	9	6
10	2	40+100=140	80+400+495=975	**14 97.5 98.5**	10	6.5
11	2	44+121=165	88+484+660=1232	**15 112 113**	11	7
12	2	48+144=192	96+576+858=1530	**16 127.5 128.5**	12	7.5
13	2	52+169=221	104+676+1092=1872	**17 144 145**	13	8
14	2	56+196=252	112+784+1365=2261	**18 161.5 162.5**	14	8.5
15	2	60+225=285	120+900+1680=2700	**19 180 181**	15	9
16	2	64+256=320	128+1024+2040=3192	**20 199.5 200.5**	16	9.5
17	2	68+289=357	136+1156+2448=3740	**21 220 221**	17	10
18	2	72+324=396	144+1296+2907=4347	**22 241.5 242.5**	18	10.5
19	2	76+361=437	152+1444+3420=5016	**23 264 265**	19	11
20	2	80+400=480	160+1600+3990=5750	**24 287.5 288.5**	20	11.5
21	2	84+441=525	168+1764+4620=6552	**25 312 313**	21	12
22	2	88+484=572	176+1936+5313=7425	**26 337.5 338.5**	22	12.5

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4	3	24+16=40	72+96+30=198	**10 49.5 50.5**	4	4.5
5	3	30+25=55	90+150+60=300	**11 60 61**	5	5
6	3	36+36=72	108+216+105=429	**12 71.5 72.5**	6	5.5
7	3	42+49=63	126+294+168=588	**13 84 85**	7	6
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15	3	90+225=315	270+1350+1680=3300	**21 220 221**	15	10
16	3	96+256=352	288+1536+2040=3864	**22 241.5 242.5**	16	10.5
17	3	102+289=391	306+1734+2448=4488	**23 264 265**	17	11
18	3	108+324=432	324+1944+2907=5175	**24 287.5 288.5**	18	11.5
19	3	114+361=475	342+2166+3420=5928	**25 312 313**	19	12
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21	3	126+441=567	378+2646+4620=7644	**27 364 365**	21	13
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3. $\mathbf{k((21_{x1})^2 + (220_{x1})^2 = (221_{x1})^2)}$ can also be derived by imagining a smaller right-triangle **xyz** with $\mathbf{y^2 = X}$ and setting the distances between **Y** and **Z**, and **x** and **z** both equal to unity**, i.e.**

$Z - Y = 1$ and $z - x = 1$, and, therefore:-

$$X = y^2 = (x+1)^2 - x^2 = 2x + 1$$

$$X^2 = (2x + 1)^2 = Z^2 - Y^2 = (Y+1)^2 - Y^2 = \underline{2Y + 1 = 4x^2 + 4x + 1}$$

$$\underline{Y = 2x^2 + 2x},\ \ \underline{Z = 2x^2 + 2x + 1},\ \text{and}\ \underline{X = 2x + 1}$$

-from which every satisfying Pythagorean square may be derived. - ***including all those with XYZ 3*** $\mathbb{Z}$ **(!)**

4. It should be noted that both this proportionality and integer field inclusion follows from the designated unitary distances and the integer coefficients of the XYZ relationships, i.e., ***if any x were irrational or complex, then, depending on the value of k chosen, at least one value (X, or Y and Z) would also have to be irrational or complex.***

5. When ***n>2***, then following the same logic outlined above (and dividing both sides of the equation containing non-unitary values of ***Z-Y*** by **Z-Y** to obtain them) :-

$$\textbf{a. } k(X^n = (Y + 1)^n - Y^n = y^{n-1} + ny^{n-2} + \ldots + ny + 1)$$

and

$$\textbf{b. } k(x^{n-1} + nx^{n-2} + \ldots + nx + 1)^n = y^{n-1} + ny^{n-2} + \ldots + ny + 1)$$

in which all solutions for y are clearly irrational or complex, as the number of nested radicals only *increases* as **n** is increased. Indeed when *any* non-zero integer value of x is input, the output for y must lie outside of $\mathbb{Q}$ **(!)** implying that *any and all* non-zero *integer inputs* of x

c. as long as k is rational cannot have rational inputs of ***y and z*** and

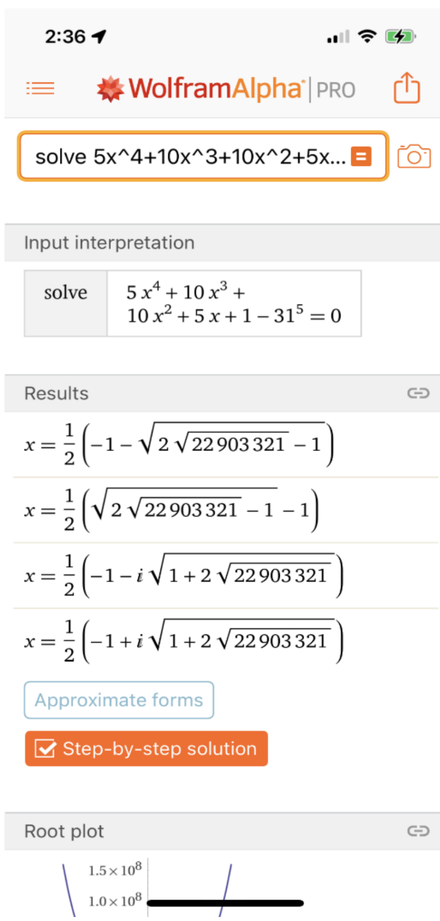
d. an irrational k implies at the very least an irrational kX .

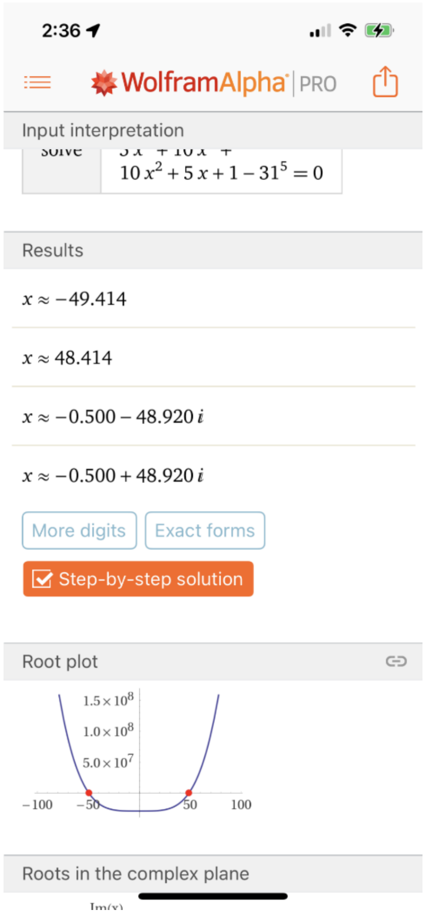
e. An example executed on **Wolfram*alpha* Pro** follows (but read y for x) for **n=5**, **y=2** and **k=1**. Here the real values are estimated to be:-

$$(X^5 =)\ 31^5 + Y^5(\sim= 48.414) = Z^5(\sim = 49.414)$$

$$28629151 + 265983581.31789303\ (= 294612732.3) \sim= 294611743.6597$$

If *any* non-zero integer value of ***x*** with unitary distances of both ***Y*** *and* ***Z*** and ***x*** *and* ***z*** yields an irrational value for ***y*** *or* ***Y***, then no rational values of all 3 variables can exist as ***X,Y, and Z cannot all be represented by algebraic integers in base x*** **.**





If this is not completely satisfying, then:-

Consider two (2) fields, "**D**" and "**F**", represented by four (4) variables, "**X**", "**Y**", "**Z**", and "**n**"over the rationals, $\mathbb{Q}$, such that neither X, nor Y nor Z are 0 (**X,Y,Z** $\neq$ **0**) and

1) "**D**" is restricted to $X^2 + Y^n = Z^2$ **AND** $Z = X + Y$
2) "**F**" is restricted to $X^n + Y^n = Z^n$

That "**D**" **and** "**F**" **are** ***mutually exclusive*** is trivial since:-

If $Z=X + Y$ and $Z^n = X^n + Y^n$, then

$$(X+Y)^n = X^n + Y^n \text{ and}$$

$$\mathbf{X^n + YX^{n-1} + \ldots + Y^{n-1}X + Y^n \;\; X \neq X^n + Y^n}$$

Now consider the number of members of "**D**" and "**F**" over the mutually distinct ranges of **n** provided by **n** ≤ 2 and **n** >2 in these mutually distinct fields:-

	n ≤ 2	**n** >2
D	{0}	{∞}*
F	{∞}	**MUST BE {0}**

x

*That "**D**" has infinite members when n>2 can be seen by setting: -

$$X = (x)((x^{n-2}-1))/2 \text{ or } ((x^{n-1}-x))/2$$

$$Y^n = ((x)((x^{n-2}+1))/2)^2 - ((x)((x^{n-2}-1))/2)^2$$

$$Z = (x)((x^{n-2}+1))/2 \text{ or } ((x^{n-1}+x))/2$$

Or, for example: -

$$\mathbf{(2^{n-2}-1)^2 + 2^n = (2^{n-2}+1)^2}$$

Or by the following tables: -

square+	cube=	square	||	square+	quartic=	square
0	0	0	||	0	0	0
0	1	1	||	0	1	1
1	2	3	||	3	2	5
3	3	6	||	12	3	15
6	4	10	||	30	4	34
10	5	15	||	60	5	65
$x(x-1)/2$			||	$x(x-1)(x+1)/2$		

square+	quintic=	square	||	square+	heptic=	square
0	0	0	||	0	0	0
0	1	1	||	0	1	1
7 `	2	9	||	15	2	17
39	3	42	||	120	3	123
126	4	130	||	510	4	514
310	5	315	||	1560	5	1565
$x(x-1)(x^2+x+1)/2$			||	$x(x-1)(x^3+x^2+x+1)/2$		