## **Fields and Fermat**

 $_{\mathbb{Z}} \mathbb{Q} \cdot \mathbb{R} \mathbb{C}$ 

-The Delinferni Fermat Theorem-

The simplest explanation why  $X^n + Y^n = Z^n$  for n > 2 cannot include all nonzero X, Y, Z  $\Im \mathbb{Z}$  or  $\Im \mathbb{Q}^n$  is that when n > 2, *X*, *Y*, and *Z* cannot all be represented by algebraic integers in base x as they can easily be when  $n \le 2$ .

1)What is it specifically about n = 2 that allows for  $x^n + y^n = z^n$  to prescribe sets which include all non-zero x, y, z 3  $\mathbb{Z}$ ?

a. For any base **x** where  $X^2+Y^2=Z^2$  and  $a=Z_0-Y_0$ 

 $(2a \cdot a_{x0}^2)^2 + (2a \cdot 2a^2 \cdot (a^3 - a)/2_{x0})^2 = (2a \cdot 2a^2 \cdot (a^3 + a)/2_{x0})^2$ 

And when  $Z_1 - Y_1 = 1$ , substituting

$$k=a^{-1}$$
 and  $x_1=x_0+(a-1)/2$ , then :-

b. 
$$k((21_{x1})^2 + (220_{x1})^2 = (221_{x1})^2)$$

2) All rational instances may therefore be inferred from:-

а	<b>x</b> 0	X <sub>0</sub> =2ax <sub>0</sub> +a^2	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1	Divisor (=a)	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2
1	3	6+1=7	18+6+0=24	7 24 25	1	3
2	3	12+4=16	36+24+3=63	8 31.5 32.5	2	3.5
3	3	18+9=27	54+54+12=120	9 40 41	3	4
4	3	24+16=40	72+96+30=198	10 49.5 50.5	4	4.5
5	3	30+25=55	90+150+60=300	11 60 61	5	5
6	3	36+36=72	108+216+105=429	12 71.5 72.5	6	5.5
7	3	42+49=63	126+294+168=588	13 84 85	7	6
8	3	48+64-80	144+384+252=780	14 97.5 98.5	8	6.5
9	3	54+81=99	162+486+360=1008	15 112 113	9	7
10	3	60+100=160	180+600+495=1275	16 127.5 128.5	10	7.5
11	3	66+121=187	198+726+660=1584	17 144 145	11	8
12	3	72+144=216	216+864+858=1938	18 161.5 162.5	12	8.5
13	3	78+169=247	234+1014+1092=2340	19 180 181	13	9
14	3	84+196=280	252+1176+1365=2793	20 199.5 200.5	14	9.5
15	3	90+225=315	270+1350+1680=3300	21 220 221	15	10
16	3	96+256=352	288+1536+2040=3864	22 241.5 242.5	16	10.5
17	3	102+289=391	306+1734+2448=4488	23 264 265	17	11
18	3	108+324=432	324+1944+2907=5175	24 287.5 288.5	18	11.5
19	3	114+361=475	342+2166+3420=5928	25 312 313	19	12
20	3	120+400=520	360+2400+3990=6750	26 337.5 338.5	20	12.5
21	3	126+441=567	378+2646+4620=7644	27 364 365	21	13
22	3	132+484=616	396+2904+5313=8613	28 391.5 392.5	22	13.5
а	x <sub>0</sub>	X <sub>0</sub> =2ax <sub>0</sub> +a^2	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1	Divisor (=a)	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2
<b>a</b> 1	<b>X</b> 0 1.5	<b>X<sub>0</sub>=2ax<sub>0</sub>+a^2</b> 3+1=4	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5	<mark>Pyth &lt;=&gt; Z₁-Y₁=1</mark> 4 7.5 8.5	Divisor (=a)	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2
<b>a</b> 1 2	<b>X</b> 0 1.5 1.5	<b>X<sub>0</sub>=2ax<sub>0</sub>+a^2</b> 3+1=4 6+4=10	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24	Pyth <=> Z₁-Y₁=1 4 7.5 8.5 5 12 13	<b>Divisor (=a)</b> 1 2	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2
<b>a</b> 1 2 3	<b>X</b> 0 1.5 1.5 1.5	<b>X<sub>0</sub>=2ax<sub>0</sub>+a^2</b> 3+1=4 6+4=10 9+9=18	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1 4 7.5 8.5 5 12 13 6 17.5 18.5	<b>Divisor (=a)</b> 1 2 3	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5
<b>a</b> 1 2 3 4	<b>X</b> 0 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1 4 7.5 8.5 5 12 13 6 17.5 18.5 7 24 25	Divisor (=a) 1 2 3 4	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3
<b>a</b> 1 2 3 4 5	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1 4 7.5 8.5 5 12 13 6 17.5 18.5 7 24 25 8 31.5 32.5	Divisor (=a) 1 2 3 4 5	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5
<b>a</b> 1 2 3 4 5 6	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1 4 7.5 8.5 5 12 13 6 17.5 18.5 7 24 25 8 31.5 32.5 9 40 41	Divisor (=a) 1 2 3 4 5 6	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4
<b>a</b> 1 2 3 4 5 6 7	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1 4 7.5 8.5 5 12 13 6 17.5 18.5 7 24 25 8 31.5 32.5 9 40 41 10 49.5 50.5	Divisor (=a) 1 2 3 4 5 6 7	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5
<b>a</b> 1 2 3 4 5 6 7 8	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480	Pyth         Z <sub>1</sub> -Y <sub>1</sub> =1           4         7.5         8.5           5         12         13           6         17.5         18.5           7         24         25           8         31.5         32.5           9         40         41           10         49.5         50.5           11         60         61	Divisor (=a) 1 2 3 4 5 6 7 8	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5
<b>a</b> 1 2 3 4 5 6 7 8 9	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5	Pyth <=> $Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5	Divisor (=a) 1 2 3 4 5 6 7 8 9	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5
<b>a</b> 1 2 3 4 5 6 7 8 9 10	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840	Pyth <=> $Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85	Divisor (=a) 1 2 3 4 5 6 7 8 9 10	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5	Pyth <=> $Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170	Pyth <=> $Z_1-Y_1=1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5	Pyth <=> $Z_1-Y_1=1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016	Pyth <=> $Z_1-Y_1=1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5         17       144       145	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5	Pyth <=> $Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5         17       144       145         18       161.5       162.5	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8 8.5
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860	Pyth <=> $Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5         17       144       145         18       161.5       162.5         19       180       181	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8 8 8.5 9
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304 51+289=340	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860 76.5+867+2448=3391.5	Pyth <=> $Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5         17       144       145         18       161.5       162.5         19       180       181         20       199.5       200.5	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8 8.5 9 9.5
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304 51+289=340 54+324=378	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860 76.5+867+2448=3391.5 81+972+2907=3960	Pyth <=> $Z_1-Y_1=1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5         17       144       145         18       161.5       162.5         19       180       181         20       199.5       200.5         21       220       221	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8 8 8.5 9 9.5 10
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304 51+289=340 54+324=378 57+361=418	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860 76.5+867+2448=3391.5 81+972+2907=3960 85.5+1083+3420=4588.5	Pyth $< > Z_1 - Y_1 = 1$ 4         7.5         8.5           5         12         13           6         17.5         18.5           7         24         25           8         31.5         32.5           9         40         41           10         49.5         50.5           11         60         61           12         71.5         72.5           13         84         85           14         97.5         98.5           15         112         113           16         127.5         128.5           17         144         145           18         161.5         162.5           19         180         181           20         199.5         200.5           21         220         221           22         241.5         242.5	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8 8 8.5 9 9.5 10 10.5
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304 51+289=340 54+324=378 57+361=418 60+400=460	$Y_0=2a^*x_0^2+2a^2^*x_0+(a^3-a)/2$ 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860 76.5+867+2448=3391.5 81+972+2907=3960 85.5+1083+3420=4588.5 90+1200+3990=5280	Pyth $< > > Z_1 - Y_1 = 1$ 4       7.5       8.5         5       12       13         6       17.5       18.5         7       24       25         8       31.5       32.5         9       40       41         10       49.5       50.5         11       60       61         12       71.5       72.5         13       84       85         14       97.5       98.5         15       112       113         16       127.5       128.5         17       144       145         18       161.5       162.5         19       180       181         20       199.5       200.5         21       220       221         22       241.5       242.5         23       264       265	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6 6.5 7 7.5 8 8 8.5 9 9.5 10 10.5 11
<b>a</b> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304 51+289=340 54+324=378 57+361=418 60+400=460 63+441=504	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860 76.5+867+2448=3391.5 81+972+2907=3960 85.5+1083+3420=4588.5 90+1200+3990=5280 94.5+1323+4620=6037.5	Pyth $< > Z_1 - Y_1 = 1$ 4         7.5         8.5           5         12         13           6         17.5         18.5           7         24         25           8         31.5         32.5           9         40         41           10         49.5         50.5           11         60         61           12         71.5         72.5           13         84         85           14         97.5         98.5           15         112         113           16         127.5         128.5           17         144         145           18         161.5         162.5           19         180         181           20         199.5         200.5           21         220         221           22         241.5         242.5           23         264         265           24         287.5         288.5	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5 5 6 6 6 6 5 7 7 7.5 8 8 8.5 9 9.5 10 10.5 11 11.5
a         1         2         3         4         5         6         7         8         9         10         11         12         13         14         15         16         17         18         19         20         21         22	<b>X</b> 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5	X <sub>0</sub> =2ax <sub>0</sub> +a^2 3+1=4 6+4=10 9+9=18 12+16=28 15+25=40 18+36=54 21+49=70 24+64-88 27+81=108 30+100=130 33+121=154 36+144=180 39+169=208 42+196=238 45+225=270 48+256=304 51+289=340 54+324=378 57+361=418 60+400=460 63+441=504 66+484=550	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2 4.5+3+0=7.5 9+12+3=24 13.5+27+12=52.5 18+48+30=96 22.5+75+60=157.5 27+108+105=240 31.5+147+168=346.5 36+192+252=480 40.5+243+360=643.5 45+300+495=840 49.5+363+660=1072.5 54+432+858=1170 58.5+507+1092=1738.5 63+588+1365=2016 67.5+675+1680=2422.5 72+768+2040=2860 76.5+867+2448=3391.5 81+972+2907=3960 85.5+1083+3420=4588.5 90+1200+3990=5280 94.5+1323+4620=6037.5 99+1452+5313=6864	Pyth $< > Z_1 - Y_1 = 1$ 4         7.5         8.5           5         12         13           6         17.5         18.5           7         24         25           8         31.5         32.5           9         40         41           10         49.5         50.5           11         60         61           12         71.5         72.5           13         84         85           14         97.5         98.5           15         112         113           16         127.5         128.5           17         144         145           18         161.5         162.5           19         180         181           20         199.5         200.5           21         220         221           22         241.5         242.5           23         264         265           24         287.5         288.5           25         312         313	Divisor (=a) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2 1.5 2 2.5 3 3.5 4 4.5 5 5 5 6 6 6.5 7 7.5 8 8 8.5 9 9.5 10 10.5 11 11.5 12

а	x <sub>0</sub>	X <sub>0</sub> =2ax <sub>0</sub> +a^2	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1	Divisor (=a)	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2
1	2	4+1=5	8+4+0=12	5 12 13	1	2
2	2	8+4=12	16+16+3=35	<mark>6 17.5 18.5</mark>	2	2.5
3	2	12+9=21	24+36+12=72	7 24 25	3	3
4	2	16+16=32	32+64+30=126	8 31.5 32.5	4	3.5
5	2	20+25=45	40+100+60=200	9 40 41	5	4
6	2	24+36=60	48+144+105=297	10 49.5 50.5	6	4.5
7	2	28+49=77	56+196+168=420	11 60 61	7	5
8	2	32+64-96	64+256+252=572	12 71.5 72.5	8	5.5
9	2	36+81=117	72+324+360=688	13 84 85	9	6
10	2	40+100=140	80+400+495=975	14 97.5 98.5	10	6.5
11	2	44+121=165	88+484+660=1232	15 112 113	11	7
12	2	48+144=192	96+576+858=1530	16 127.5 128.5	12	7.5
13	2	52+169=221	104+676+1092=1872	17 144 145	13	8
14	2	56+196=252	112+784+1365=2261	18 161.5 162.5	14	8.5
15	2	60+225=285	120+900+1680=2700	19 180 181	15	9
16	2	64+256=320	128+1024+2040=3192	20 199.5 200.5	16	9.5
17	2	68+289=357	136+1156+2448=3740	21 220 221	17	10
18	2	72+324=396	144+1296+2907=4347	22 241.5 242.5	18	10.5
19	2	76+361=437	152+1444+3420=5016	23 264 265	19	11
20	2	80+400=480	160+1600+3990=5750	24 287.5 288.5	20	11.5
21	2	84+441=525	168+1764+4620=6552	25 312 313	21	12
22	2	88+484=572	176+1936+5313=7425	26 337.5 338.5	22	12.5
а	<b>x</b> <sub>0</sub>	X <sub>0</sub> =2ax <sub>0</sub> +a^2	Y <sub>0</sub> =2a*x <sub>0</sub> ^2+2a^2*x <sub>0</sub> +(a^3-a)/2	Pyth <=> Z <sub>1</sub> -Y <sub>1</sub> =1	Divisor (=a)	x <sub>1</sub> =>x <sub>0</sub> +(a-1)/2
1	3	6+1=7	18+6+0=24	7 24 25	1	3
2	3	12+4=16	36+24+3=63	8 31.5 32.5	2	3.5
3	3	18+9=27	54+54+12=120	9 40 41	3	4
4	3	24+16=40	72+96+30=198	10 49.5 50.5	4	4.5
5	3	30+25=55	90+150+60=300	11 60 61	5	5
6	3	36+36=72	108+216+105=429	12 71.5 72.5	6	5.5
7	3	42+49=63	126+294+168=588	13 84 85	7	6
8	3	48+64-80	144+384+252=780	14 97.5 98.5	8	6.5
9	3	54+81=99	162+486+360=1008	15 112 113	9	7
10	3	60+100=160	180+600+495=1275	16 127.5 128.5	10	7.5
11	3	66+121=187	198+726+660=1584	17 144 145	11	8
12	3	72+144=216	216+864+858=1938	18 161.5 162.5	12	8.5
13	3	78+169=247	234+1014+1092=2340	19 180 181	13	9
14	3	84+196=280	252+1176+1365=2793	20 199.5 200.5	14	9.5
15	3	90+225=315	270+1350+1680=3300	21 220 221	15	10

3.  $k((21_{x1})^2 + (220_{x1})^2 = (221_{x1})^2)$  can also be derived by imagining a smaller right-triangle xyz with  $y^2 = X$  and setting the distances between Y and Z, and x and z both equal to unity, i.e.

22 241.5 242.5

**23 264 265** 

24 287.5 288.5

25 312 313

26 337.5 338.5

28 391.5 392.5

27 364 365

16

17

18

19

20

21

22

10.5

11

11.5

12

12.5

13

13.5

16

17

18

19

20

21

22

3

3

3

3

3

3

3

96+256=352

102+289=391

108+324=432

114+361=475

120+400=520

126+441=567

132+484=616

288+1536+2040=3864

306+1734+2448=4488

324+1944+2907=5175

342+2166+3420=5928

360+2400+3990=6750

378+2646+4620=7644

396+2904+5313=8613

**Z** -  $\mathbf{Y} = \mathbf{1}$  and  $\mathbf{z} - \mathbf{x} = \mathbf{1}$ , and, therefore:-

$$X = y^{2} = (x+1)^{2} - x^{2} = 2x + 1$$
$$X^{2} = (2x+1)^{2} = Z^{2} - Y^{2} = (Y+1)^{2} - Y^{2} = \underline{2Y+1} = 4x^{2} + 4x + 1$$
$$\underline{Y = 2x^{2} + 2x}, \quad \underline{Z = 2x^{2} + 2x + 1}, \text{ and } \underline{X = 2x + 1}$$

-from which every satisfying Pythagorean square may be derived. including all those with XYZ 3  $\mathbb{Z}$  (!)

4. It should be noted that both this proportionality and integer field inclusion follows from the designated unitary distances and the integer coefficients of the XYZ relationships, i.e., *if any x were irrational or complex, then, depending on the value of k chosen, at least one value (X, or Y and Z) would also have to be irrational or complex.* 

5. When <u>n>2</u>, then following the same logic outlined above (and dividing both sides of the equation containing non-unitary values of *Z*-*Y* by *Z*-*Y* to obtain them) :-

a. 
$$k(X^n = (Y + 1)^n - Y^n = y^{n-1} + ny^{n-2} + ... + ny + 1)$$

and

**b.** 
$$k(x^{n-1} + nx^{n-2} + ... + nx + 1)^n = y^{n-1} + ny^{n-2} + ... + ny + 1)$$

in which all solutions for y are clearly irrational or complex, as the number of nested radicals only *increases* as **n** is increased. Indeed when <u>any</u> non-zero integer value of x is input, the output for y must lie outside of  $\mathbb{Q}$  (!) implying that any and all non-zero integer inputs of x

c. as long as k is rational cannot have rational inputs of y and z and

**d**. an irrational k implies at the very least an irrational kX.

e. An example executed on Wolfram*alpha* Pro follows (but read y for x) for n=5, y=2 and k=1. Here the real values are estimated to be:-

$$(X^5 =) 31^5 + Y^5( = 48.414) = Z^5( = 49.414)$$

**28629151** + **265983581.31789303** (= 294612732.3) ~= **294611743.6597** 

If *any* non-zero integer value of x with unitary distances of both Yand Z and x and z yields an irrational value for y or Y, then no rational values of all 3 variables can exist as <u>X,Y, and Z cannot</u> all be represented by algebraic integers in base x.

2:36 🕇	.ull 🗢	<b>1</b>			
≔ <b>∦</b> W	olframAlpha <sup>-</sup>  PRO	ᠿ			
solve 5x^4+	+10x^3+10x^2+5x 🗖	<b>1</b> 07			
Input interpret	ation				
solve 5 x 10	$x^{4} + 10x^{3} + x^{2} + 5x + 1 - 31^{5} = 0$				
Results		Ð			
$x = \frac{1}{2} \left( -1 - \sqrt{1 - 1} \right)$	$\left(2\sqrt{22903321}-1\right)$				
$x = \frac{1}{2} \left( \sqrt{2} \sqrt{2} \right)^2$	22903321 - 1 - 1				
$x = \frac{1}{2} \left( -1 - i \right)^2$	$\sqrt{1+2\sqrt{22903321}}$				
$x = \frac{1}{2} \left( -1 + i \right)^2$	$\sqrt{1+2\sqrt{22903321}}$				
Approximate forms					
Step-by-s	tep solution				
Root plot		Ð			
$1.5 \times 10^8$ $1.0 \times 10^8$	/				

2:36 <b>1</b> 🗧	<b>?</b> ₩					
📰 🐐 Wolfram Alpha   pro	ᠿ					
Input interpretation Solve $3x + 10x + 10x + 10x^2 + 5x + 1 - 31^5 = 0$						
Results						
<i>x</i> ≈ −49.414						
<i>x</i> ≈ 48.414						
$x \approx -0.500 - 48.920 i$	$x \approx -0.500 - 48.920 i$					
$x \approx -0.500 + 48.920 i$						
More digits Exact forms						
Step-by-step solution						
Root plot	Ð					
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						
Roots in the complex plane						
Im(v)						

If this is not completely satisfying, then:-

Consider two (2) fields, "**D**" and "**F**", represented by four (4) variables, "**X**", "**Y**", "**Z**", and "**n**" over the rationals,  $\mathbb{Q}$ , such that neither X, nor Y nor Z are 0 (**X**,**Y**,**Z**  $\neq$  **0**) and

1) "D" is restricted to  $X^2 + Y^n = Z^2$  AND Z = X + Y

2) "**F**" is restricted to  $X^n + Y^n = Z^n$ 

That "D" and "F" are mutually exclusive is trivial since:-

If 
$$Z=X + Y$$
 and  $Z^n = X^n + Y^n$ , then

## $(X+Y)^n = X^n + Y^n$ and

## $X^{n} + YX^{n-1} + ... + Y^{n-1}X + Y^{n} \quad X \neq X^{n} + Y^{n}$

Now consider the number of members of "**D**" and "**F**" over the mutually distinct ranges of **n** provided by  $n \le 2$  and n > 2 in these mutually distinct fields:-

	$\mathbf{n} \leq 2$	<b>n</b> >2
D	{0}	$\{\infty\}^*$
F	$\{\infty\}$	MUST BE {0}
	X	

\*That "**D**" has infinite members when n>2 can be seen by setting: -

 $X = (x)((x^{n-2}-1))/2 \text{ or } ((x^{n-1}-x))/2$  $Y^{n} = ((x)((x^{n-2}+1))/2)^{2} - ((x)((x^{n-2}-1))/2)^{2}$  $Z = (x)((x^{n-2}+1))/2 \text{ or } ((x^{n-1}+x))/2$ 

Or, for example: -

 $(2^{n-2}-1)^2 + 2^n = (2^{n-2}+1)^2$ 

Or by the following tables: -

square+	cube=	square	П	square+	quartic=	square
			=  =:			
0	0	0	П	0	0	0
0	1	1	П	0	1	1
1	2	3	П	3	2	5
3	3	6	П	12	3	15
6	4	10	П	30	4	34
10	5	15	П	60	5	65
x(x-1)/2			П	x(x-1)(x+1)/2		
===========	=============		=  =:	============	=============	

square+ quintic= square square+ heptic= square 0 0 0 0 0 0 1 1 || 0 1 0 1 7 2 9 || 15 2 17 123 3 42 || 120 3 39 126 4 130 510 4 514 315 | 1560 5 1565 310 5  $x(x-1)(x^2+x+1)/2$  $|| x(x-1)(x^3+x^2+x+1)/2$ ------