## A Non-Abstract Algebraic Proof of Fermat's Last Theorem

 By DM Delinfernia, Jr.The only universal restriction to the set forming Pythagorean Triples (of the $(\mathrm{n}=2)$ second order) can be represented by the following equivalent identities labeled as (\#1) and (\#2) respectively:-

For any base $\mathbf{x}$ where $\mathbf{X}_{0}{ }^{2}+\mathbf{Y}_{0}{ }^{2}=\mathbf{Z}_{0}{ }^{2}$ and $\mathbf{a}=\mathbf{Z}_{0}-\mathbf{Y}_{0}$
\#1) $\left(2 a x_{0}+a^{2}\right)^{2}+\left(2 a x_{0}{ }^{2}+2 a^{2} x_{0}+\left(a^{3}-a\right) / 2\right)^{2}=\left(2 a x_{0}{ }^{2}+2 a^{2} x_{0}+\left(a^{3}-a\right) / 2\right)^{2}$
Consider two right-triangles, A and B (Figure 1) which are related in that

a) the base of the larger triangle $(\mathbf{B})$ is the square of the base of the smaller triangle (A)
b) the difference between the hypotenuse and the vertical arms of both triangles is the same and equal to some measure, "a"
c) we will be interested in framing the vertical arms, " $x$ " and " $y$ ", of the both triangles, (A) and (B), as functions of the vertical arm, " $d$ ", of the smaller triangle, (A), as well as of the difference, "a".

$$
\begin{gathered}
\mathrm{x}=\left(\mathrm{a}=\mathrm{x}^{1 / 2}\right)^{2}=(\mathrm{d}+\mathrm{a})^{2}-\mathrm{d}^{2}=\left(2 \mathrm{ad}+\mathrm{a}^{2}\right)=2 \underline{2 a \cdot a^{2} \underline{d}} \\
\mathrm{x}^{2}=(\mathrm{y}+\mathrm{a})^{2}-\mathrm{y}^{2}=\left(2 \mathrm{ay}+\mathrm{a}^{2}\right)=\left(2 \mathrm{ad}+\mathrm{a}^{2}\right)^{2}=4 \mathrm{a}^{2} \mathrm{~d}^{2}+4 \mathrm{a}^{3} \mathrm{~d}+\underline{a^{4}} \\
2 \mathrm{ay}=4 \mathrm{a}^{2} \mathrm{~d}^{2}+4 \mathrm{a}^{3} \mathrm{~d}+\mathrm{a}^{4}-\mathrm{a}^{2} \\
\mathrm{y}=2 \mathrm{ad}^{2}+2 \mathrm{a}^{2} \mathrm{~d}+\left(\mathrm{a}^{3}-\mathrm{a}\right) / 2=\underline{2 a \cdot 2 a^{2} \cdot\left(a^{3}-a\right) / 2} \underline{d} \\
\mathrm{z}=\mathrm{y}+\mathrm{a}=\underline{2 a \cdot 2 a^{2} \cdot\left(a^{3}+a\right) / 2} \underline{d}
\end{gathered}
$$

And if $\mathbf{Z}_{1}-Y_{1}=\mathbf{1}$, substituting $k=\mathbf{a}^{-1}$ and $\mathbf{x}_{1}=\mathbf{x}_{\mathbf{0}}+(\mathbf{a}-\mathbf{1}) / \mathbf{2}$, then :-
\#2) $k\left[\left(2 x_{1}+1\right)^{2}+\left(2 x_{1}{ }^{2}+2 x_{1}\right)^{2}=\left(2 x_{1}{ }^{2}+2 x_{1}+1\right)^{2}\right]$
or where subscripts represent the base variable "x $\mathbf{x}_{1}$ " and "." separates its powers,

$$
\text { \#2a) } \mathbf{k}\left[\left(2 \cdot 1_{\mathrm{x} 1}\right)^{2}+\left(2 \cdot 2 \cdot 0_{\mathrm{x} 1}\right)^{2}=\left(\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{1}_{\mathrm{x} 1}\right)^{2}\right]
$$

Then, if some $\mathbf{X}_{1}{ }^{\mathbf{n}}+\mathbf{Y}_{\mathbf{1}}{ }^{\mathbf{n}}=\mathbf{Z}_{\mathbf{1}}{ }^{\mathbf{n}}$ were to represent another true identity, it would of necessity map to any other true identity including, but not limited to, $\mathbf{X}_{0}{ }^{2}+\mathbf{Y}_{0}{ }^{2}=\mathbf{Z}_{0}{ }^{2}$. So, dividing both sides of $\mathbf{X}_{1}{ }^{\mathrm{n}}+\mathbf{Y}_{1}{ }^{\mathbf{n}}=\mathbf{Z}_{1}{ }^{\mathrm{n}}$ by $\mathbf{k}=\left(\mathbf{Z}_{1}-\mathbf{Y}_{1}\right)$, and letting $\mathbf{A}=\mathbf{X}_{1} / \mathbf{k}, \mathbf{B}=\mathbf{Y}_{1} / \mathbf{k}$, and $\mathbf{C}=\mathbf{Z}_{1} / \mathbf{k}$, then $(\mathbf{C}-\mathbf{B})=\mathbf{1}$, and :-

$$
\mathbf{k}\left(\mathbf{A}^{\mathrm{n}}+\mathbf{B}^{\mathrm{n}}=\mathbf{C}^{\mathrm{n}}\right)
$$

Therefore via identity \#2a

$$
k\left[\left({ }^{n} \sqrt{ }\left(2 \cdot 1_{x 1}\right)^{2}\right)^{n}+\left({ }^{n} \sqrt{ }\left(2 \cdot 2 \cdot 0_{x 1}\right)^{2}\right)^{n}=\left({ }^{n} \sqrt{ }\left(2 \cdot 2 \cdot 1_{x 1}\right)^{2}\right)^{n}\right]
$$

substituting $\mathbf{A}={ }^{\mathrm{n}} \sqrt{ }\left(\mathbf{2} \cdot \mathbf{1}_{\mathbf{x} 1}\right)^{\mathbf{2}}, \mathbf{B}=\mathrm{n} \sqrt{ }\left(\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{0}_{\mathbf{x}}\right)^{\mathbf{2}}$ and $\mathbf{C}=\mathrm{n} \sqrt{ }\left(\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{0}_{\mathbf{x} 1}\right)^{\mathbf{2}}$, then in order for $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ to all be non-zero and rational with any odd $n>2$, all three would of necessity have to have rational $\mathbf{2 n}^{\text {th }}$ roots - which were which seems on its face to be absurd as B and C differ by 1 . This is confirmed via a very nice proof offered by Professor A. Rosenbloom of the University of Toronto, since $B$ and $C$ differ by 1 , and $B=Y^{2 n}$, and $C=B+1=Z^{2 n}$, then $1=Z^{2 n}$ - $Y^{\mathbf{2 n}}$ and $\mathbf{1}=\left(\mathbf{Z}^{\mathrm{n}}+\mathbf{Y}^{\mathrm{n}}\right)\left(\mathbf{Z}^{\mathrm{n}}-\mathbf{Y}^{\mathrm{n}}\right)$. Therefore both $\left(\mathbf{Z}^{\mathrm{n}}+\mathbf{Y}^{\mathrm{n}}\right)$ and ( $Z^{\mathrm{n}}-\mathbf{Y}^{\mathrm{n}}$ ) must divide and equal 1 implying either $\mathbf{Z}$ or $Y=0$ and the other $=1$.

And, therefore, FLT must hold for non-zero rationals with any odd $\mathbf{n}>2$.

